Brang et al. *EPJ Quantum Technology* (2024) 11:33
https://doi.org/10.1140/epjqt/s40507-024-00244-y

EPJ Quantum Technology
a SpringerOpen Journal

RESEARCH

Open Access

Spooky action at a distance? A two-phase study into learners' views of quantum entanglement

Michael Brang[1], Helena Franke[1], Franziska Greinert[2], Malte S. Ubben[2], Fabian Hennig[3] and Philipp Bitzenbauer[3*]

*Correspondence:
philipp.bitzenbauer@fau.de
[3]Didaktik der Physik, Department Physik,
Friedrich-Alexander-Universität Erlangen-Nürnberg, Staudstr. 7, Erlangen, 91058, Germany
Full list of author information is available at the end of the article

Abstract

Quantum entanglement is a challenging concept within the field of physics education, often eluding a full grasp by both educators and learners alike. In this paper, we report findings from a two-phase empirical study into the views of entanglement held by pre-service physics teachers and physics students from various universities. In the first phase, we utilized a questionnaire consisting of open-ended questions which was completed by 31 pre-service physics teachers. The study participants' ideas were explored using qualitative content analysis which led to the creation of rating scale items used in study phase 2. These items were administered to a broader cohort including 73 physics university students in order to capture the learners' agreement or disagreement with the questionnaire statements, and hence, helped to validate and substantiate the in-depth insights from study phase 1. Key findings revealed widespread accurate notions, like the need to consider the entire system when examining entangled states. However, less elaborated views were also identified, including ideas such as that measurements of entangled states always show perfect (anti-)correlation. Another striking observation was the confusion between quantum entanglement and superposition. In the case of quantum teleportation, many participants seemed to have a basic grasp of the concept, although a number of misconceptions were apparent, notably the idea that quantum entanglement enables faster-than-light communication. Practically, the findings can assist educators in anticipating and addressing widespread (mis-)conceptions, paving the way for more effective instruction in quantum mechanics and its real-world applications, such as quantum cryptography and computing.

Keywords: Quantum physics; Entanglement; Teleportation; Empirical study

1 Introduction

Quantum entanglement is one of the fundamental quantum concept relevant to advances in second-generation quantum technologies (QTs) [1, 2]. The use of entanglement enables a new kind of parallelism within quantum computing and, hence, a speedup with regards to solving special computational problems [3], as well as physically secure communication using entanglement in quantum teleportation [4, 5]. In addition, the 2022 Nobel Prize "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science" awarded to Alain Aspect, John Clauser and Anton Zeilinger [6] has brought public attention to the topic.

With these technologies gaining industrial relevance, there is a growing need for a well-educated quantum workforce as well as a quantum aware general public [7–9]. In Europe, efforts to train the future quantum workforce are driven by the Quantum Flagship [10], which provides the European Competence Framework for Quantum Technologies [11] and supports efforts such as the development of master's programs within the DigiQ project [12] or the development of industrial training in QTIndu [13].

However, today's quantum physics education usually starts in school. While entanglement is currently not widely present in school curricula [14], it is expected to make its way into syllabi with the increasing relevance of QT in industry. At the same time, quantum physics and especially quantum entanglement have often been considered incomprehensible or "spooky". For the above reasons, the views of entanglement held by pre-service physics teachers are of particular relevance when designing instructional sequences, physics teacher training courses or lecture series. The latter objective is not only interesting for pre-service physics teachers but, more general, for today's physics learners in schools and beyond.

This is where this paper comes in: We report the findings of a two-phase empirical study into learners' views of quantum entanglement. In phase 1, we used a questionnaire consisting of open-ended questions to explore pre-service physics teachers views of quantum entanglement. To substantiate the findings from phase 1, we used a questionnaire in phase 2 consisting of statements exhibiting views that emerged from the first survey round results. We administered the questionnaire to physics university students to rate their degree of (dis-)agreement with these statements on a rating scale. Details of the study design can be found in Sect. 4. The results are presented in Sect. 5, followed by a discussion in Sect. 6. We provide implications for both educational research and practice based on our findings in Sect. 7. In the next Sect. 2, we provide an overview of the theoretical background underlying our study.

2 Theoretical background

2.1 Quantum entanglement and teleportation: a brief overview

A qubit or quantum bit is the backbone of quantum information science and technologies and the fundamental building block for quantum computing and communication. The computational qubit with basis states $|0\rangle$ and $|1\rangle$ is an abstraction from any kind of physical realization, e.g., the polarisation states of photons or spin states of electrons. For an extended introduction into quantum concepts like superposition or entanglement, in particular using the Dirac notation we refer the reader to Refs. [15, 16].

The general state of a qubit is a superposition state of the basis states $|0\rangle$ and $|1\rangle$:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{1}$$

with $|\alpha|^2 + |\beta|^2 = 1,\ \alpha, \beta \in \mathbb{C}$. For an entangled quantum state, at least (and in the easiest case) two qubit states $|\psi_1\rangle$ and $|\psi_2\rangle$ are required. In cases where two-qubit systems are in a state that can be written as the product of the two individual qubits, i.e., $|\psi_{12}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, the qubits are referred to as separable or unentangled. In contrary, if the overall state can not be expressed as a product of the individual states, the qubits are referred to as entangled. Consequently, to describe a system of entangled qubits the whole system has to be considered, not its parts alone: Since the parts themselves can not be described individually, measurements on entangled states will lead to correlations in the outcomes.

Popular entangled states are the so-called Bell states

$$\left|\Phi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle \pm |11\rangle\big) \quad \text{and} \tag{2}$$

$$\left|\Psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\big(|01\rangle \pm |10\rangle\big), \tag{3}$$

where $|01\rangle = |0\rangle_1 \otimes |1\rangle_2$ is a short notion referring to the states of the two qubits involved. These four Bell states are maximally entangled and measurements of the respective observable would lead to perfect (anti-)correlation. For example, if two qubits are prepared in the state $|\Phi^+\rangle$ and measured in the 01-basis, one qubit will always be measured in the same state as the other, either $|0\rangle$ or $|1\rangle$: Though it is not predictable which of these states one specific qubit takes, after the measurement of one state the other can be predicted to be measured in the same state.

Quantum teleportation describes the process of "teleporting" the state of one quantum object to another quantum object using an entangled pair of quantum objects, e.g., photons [17]. The first experimental realization of quantum teleportation by the Zeilinger group [18] dates back to 1997.

For quantum teleportation, three quantum objects are required. The first quantum object S ("state to send") contains the information that should be transferred in form of a one-qubit state, e.g., $|\psi_S\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$. The further two quantum objects A and B are in an entangled state, e.g. the Bell state $\left|\Phi^+_{AB}\right\rangle$, and have to be shared between a sender A (commonly referred to as "Alice") and a receiver B ("Bob"). Therefore the overall state of the three objects becomes

$$|\psi_{SAB}\rangle = |\psi_S\rangle \otimes \left|\Phi^+_{AB}\right\rangle. \tag{4}$$

First, the sender Alice has to make the first quantum object interact with one of the two quantum objects of the entangled pair: a so-called Bell measurement has to be conducted. Hereby, quantum objects S and A are measured to be in one of the Bell states (equations (2), (3)). Consequently, this leaves the overall state $|\psi_{SAB}\rangle$ to be either of the following:

$$\left|\Phi^+_{SA}\right\rangle \otimes \big(\alpha\,|0\rangle_B + \beta\,|1\rangle_B\big) \quad \text{or} \tag{5}$$

$$\left|\Phi^-_{SA}\right\rangle \otimes \big(\alpha\,|0\rangle_B - \beta\,|1\rangle_B\big) \quad \text{or} \tag{6}$$

$$\left|\Psi^+_{SA}\right\rangle \otimes \big(\alpha\,|1\rangle_B + \beta\,|0\rangle_B\big) \quad \text{or} \tag{7}$$

$$\left|\Psi^-_{SA}\right\rangle \otimes \big(\alpha\,|1\rangle_B - \beta\,|0\rangle_B\big) \tag{8}$$

Based on the measurement results, a specific, physically easy interaction with quantum object B (a so-called Pauli-transformation) is necessary in order for Bob to receive state $|\psi_B\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$. To conduct this, classical information needs to be transmitted from Alice to Bob, to convey which measurement result Alice observed and thus which manipulation is needed. For that reason, faster-than-light transmission of information is not possible. Finally, the third quantum object B remains in the initial state of the first object S, while it is not possible to reconstruct this state at the first object. For a detailed presentation see e.g. [2, 16].

2.2 Physics education research on entanglement

Physics Education Research (PER), amongst other things, uncovers how learners understand physics topics and concepts and how to support them in their learning processes [19–21]. Particularly, research on learners' conceptions of topics in quantum physics, has become a topic of interest in PER in recent years [22–27]. It is evident that quantum concepts like quantum entanglement are frequently linked with mysticism and science fiction [28], and students tend to be unaware of applications of quantum physics concepts in their everyday lives [29]. This calls for the design of educational approaches to enlighten and instruct students in quantum physics in general and regarding topics such as quantum entanglement in particular [30].

Over the past few decades, diverse teaching strategies on how to introduce quantum mechanics in secondary or higher education settings have emerged from PER, some of which have been evaluated empirically [28, 31–38]. Although some courses include the concepts of quantum entanglement and non-locality, most of them do not include entanglement as a core content. For instance, 20 years ago Müller and Wiesner [38], who explicitly mentioned the relevance of quantum entanglement, noted that the concept was not included in their secondary school teaching concept due to time constraints. Apart from teaching concepts and educational paths themselves, real experiments [39–42], simulations [43] and interactive screen experiments [44, 45] that can be used to enrich learning environments focusing on quantum entanglement have been brought forth.

Also, educational games aimed at fostering students' understanding of quantum entanglement have been developed [46–49]: The educational game Quantum Tic-Tac-Toe [48] is a game built around the classical game of Tic-Tac-Toe where different levels of "quantumness" are employed. There exists an online application for direct classroom use [50]. This application has been used in the context of an intervention study on the high school level, and has been shown to have a positive effects on student understanding [36].

Chiofalo et al. [36] analysed conversations during data collection in which students explained the concept of quantum entanglement to their classmates. The authors found that 75% of the study participants believed that entanglement is being characterized by quantum objects not being independent of each other but their measurement to be correlated. Additionally, 15% of the participants understood entanglement as the fact that properties can not be assigned to an individual entangled quantum object. Only once they are measured, individual properties can be determined as the subsystems are no longer entangled. A tenth of the participants used 'entanglement' in equivalent to the term 'superposition'. The confusion of these two concepts became even more apparent when the subjects were asked to explain the difference. Here, 30% of the students confused quantum entanglement with superposition instead of clearly distinguishing them.

Kohnle and Deffebach [51] conducted a study exploring misconceptions of entanglement held by university students. Up to 35% of the participants were found to have the idea that—in the case of pairs of entangled quanta—the measurement of one quantum object always completely determines the measurement result of the second quantum object. Thus, according to the learners asked, the measurements on entangled quantum objects will always show perfect (anti-)correlations. This is in fact not in line with the scientific view, as the degree of correlation is dependent on the observable measured. However, it is emphasized that learners who only talk about perfect correlation need to explore the transition from maximal to non-maximal entanglement. Simulations of quantum entanglement are recommended for this purpose [43]. Further misconceptions uncovered in their study are associated with the mathematical description and physical interpretation of quantum states. In their conclusions, the authors also point to the confusion of entanglement and superposition observed by Chiofalo et al. [36].

3 Research question

As shown in the previous section, PER has brought forth numerous educational proposals for teaching the topic of entanglement. In the sense of the Model of Educational Reconstruction, however, the subject and the learner perspective have to be considered equally in the design of learning environments—in a nutshell: a solid understanding of learners' view of entanglement may inform the development of instructional materials on the topic under investigation tailored to the learners' needs. However, the systematic exploration of learners' qualitative conceptions of quantum entanglement is a research desideratum to date.

In this article, we touch upon the identified research desideratum by posing the following research question:

What views do learners (i.e., pre-service physics teachers and university physics students) have of quantum entanglement with regards to...

(i) *...entangled sates?*
(ii) *...quantum teleportation?*
(iii) *...applications?*

We justify the selection of these three specific domains (i) to (iii) through (a) their relevance for or connection to current (secondary school) curricula and (b) their representation in previous physics education research and development helping us to contextualize our study within the existing body of literature.

4 Methods

4.1 Study design and sample

To answer the research question, a two-stage study has been conducted (see Fig. 1). The first part of the study was designed as a questionnaire survey. The questionnaire consisted of open-ended questions (for a detailed description of the instruments see the following Sects. 4.2.1 and 4.2.2) and was administered to N = 31 pre-service physics teachers who had successfully completed training on theoretical and experimental quantum physics before, which not necessarily included courses about quantum entanglement. Based on the results of phase 1 of the study, a second questionnaire consisting of rating scale items was created. While in phase 1 of the study, we aimed at an in-depth exploration of learners' views of quantum entanglement, the goal of this second study phase was to substantiate and validate the findings of phase 1. Therefore, in this second study phase a larger sample (N= 73; male 55, female 13, no answer 5) was used. The cohort consisted of different physics students: 34 pre-service physics teachers, 14 bachelor's physics students, 14 master's physics students and 12 others - including engineering students or students with physics being their minor subject.

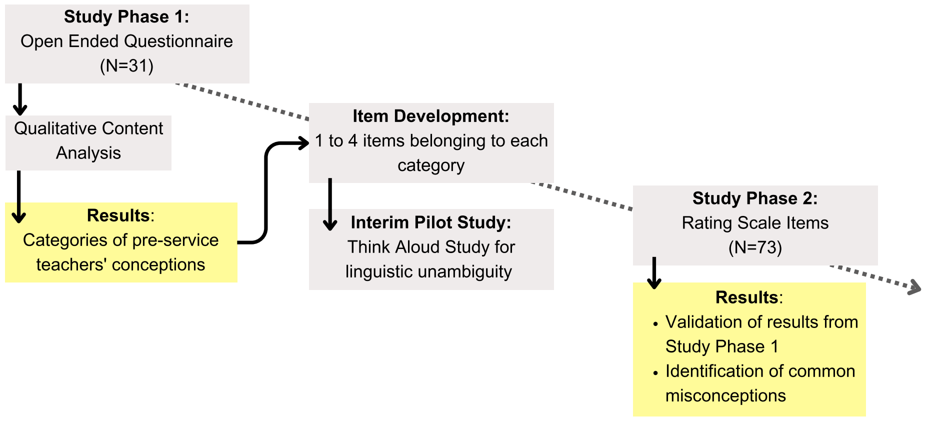

Figure 1 Overview of two phase study design including methodology and types of results

Table 1 Questions of study phase 1

Aspect	Corresponding questions included in the questionnaire
(i) Entangled States	Q1: Explain (either qualitatively, mathematically, or in both ways) how entangled quantum states can be represented using at least one specific example. Q2: Explain the representation of entangled quantum states from the previous question using at least one concrete example (qualitative, mathematical or both).
(ii) Quantum teleportation	Q3: The Nobel Prize in physics in 2022 was awarded to three researchers who have worked on entanglement, including Anton Zeilinger, who experimentally investigated the effect of quantum teleportation. Outline why—in your opinion—the term teleportation is used in connection with quantum entanglement. What does teleportation mean in this context?
(iii) Application	Q4: "Quantum entanglement follows directly from the formalism of quantum physics. It, therefore, has theoretical implications, but no scientific or technical relevance for applications." Evaluate and judge the above statement in detail.

4.2 Instruments

4.2.1 *Instrument used in study phase* 1

We developed a questionnaire to explore the study participants' views of the different aspects of entanglement (see research question in Sect. 3). We used open-ended questions in this study phase to gain in-depth insights into the pre-service physics teachers' conceptions rather than into superficial ideas. An overview of the questions included in the questionnaire and the corresponding aspects touched can be found in Table 1.

The instrument was administered as an online questionnaire implemented via the SosciSurvey tool (https://www.soscisurvey.de). It is to be noted that both questionnaires were designed and used in the German language. All questions, items and examples provided here were translated for this paper.

4.2.2 *Instrument used in study phase* 2

The categories that have emerged from the analysis of data gathered in study phase 1 (for details on the data analysis see Sect. 4.3.1) served as a basis for the creation of rating scale items used in study phase 2. For this purpose, 1 to 4 statements were created for each of the categories that emerged from the student responses regarding questions associated to the three content domains (i) entangled states, (ii) quantum teleportation and (iii) applications (as defined in the research question in Sect. 3). A detailed list of all statements created can be found in Tables 7 (regarding domain i), 8 (regarding domain ii), and 9 (regarding domain iii). The study participants in study phase 2 were then asked to rate their degree of agreement with these statements on a 5-point rating scale (where 1 corresponds to "totally disagree", and 5 to "totally agree").

Prior to its use in the main study, the questionnaire was evaluated in a qualitative pilot study. In this pilot study, we independently asked two physics students to think aloud while working on the questionnaire items. We used the students thoughts to gain insights into how the items where understood in terms of content and to identify options to improve the items in terms of language. Therefore, besides making the students work through all items, they were asked to paraphrase all items in their own words fist. Any points of misunderstanding led to the re-formulation of the items and the think aloud procedure was then repeated with the same students to check if the item formulations had improved. This process was repeated until both test persons could correctly rephrase the statements and did not voice any further misunderstandings.

In the main study, the instrument was again administered as an online questionnaire implemented via the SosciSurvey tool (https://soscisurvey.de).

4.3 Data analysis

4.3.1 Analysis carried out on study phase 1 *data*

Qualitative content analysis was applied to analyze the student responses to the individual questions of the instrument and the categories were formed inductively based on the data [52]. We present the resulting category systems including coding instructions and anchoring examples in the results section of this article. In the coding process, all categories were treated equally, and any repetitive occurrences of the same category in a participant's response were not coded, following similar research [53], as they did not provide any new insights into the participants' ideas. Finally, frequency analysis was conducted to count the number of occurrences of all categories. All participants' answers were coded by two independent raters, and Cohen's κ was calculated as a measure of the degree of the raters' agreement. Cohen's κ values for the categorizations ranged from $\kappa = 0.70$ to $\kappa = 0.85$ for the different questions, which corresponds to substantial to almost perfect agreement according to Ref. [54].

4.3.2 Analysis carried out on study phase 2 *data*

The participants' ratings of the various statements that were part of the study phase 2 instrument are presented globally using diverging stacked bar charts. We provide descriptive statistics such as mean values μ and standard deviations SD for the agreement ratings regarding all items. Hence, a higher mean value corresponds to a higher level of agreement with the respective item among the study cohort.

In the course of further data analysis, we did not classify student ratings as either "true" or "false". In contrast, we classified them to be in line or not in line with the current the scientific views. We are aware of the different interpretations of quantum physics that make it difficult to judge what the current scientific view of some topics actually is, in particular with regards to interpretation. Hence, we referred to the widely accepted Copenhagen interpretation of quantum physics and ratings are considered to align with current scientific view they align with this interpretation. We refrained from the classification of student ratings into "in line with scientific view" or "not in line with the scientific view" for the items of domain (iii), as these items refer to applications of quantum technologies today and, in particular, in the future. We provide an overview of all items and their judgments in Tables 7 to 9.

We report the correlation between the participants' ratings on the different items to explore relationships among them. We used Spearman's ρ coefficient since the data collected were of ordinal scale. Following [55], correlations $|\rho| < 0.20$ were considered weak, $0.20 \leq |\rho| \leq 0.30$ as medium, and $|\rho| > 0.30$ indicated strong correlation.

5 Results

5.1 Study phase 1 results

5.1.1 Conceptions of quantum entanglement with regards to entangled states

An overview of the conceptions found regarding entangled states is given in the category system in Table 2, alongside coding rules, anchoring examples and occurrence frequencies. The first two questions of the questionnaire dealt with the description of entangled states (question Q1, see Table 1) and asked for an explicit example (question Q2, see Table 1) and, hence, provided comprehensive insights into the pre-service physics teachers' views of entangled quantum states.

Eight of the study participants described entanglement as a property of a composite system of quantum objects that cannot be described independently of each other, but only have meaning together (category 1). This was externalised, for example, in statements declaring that there is only one common state of the whole system. Another type of statement in this category was the following:

> *"Two quanta (e.g. two leptons like the electron) are entangled if they contain coupled information."* (Participant 7)

Here the property of entanglement as a description of a system as a whole is encoded in the phrase *"coupled information"*.

Interestingly, many of the answers related to the measurement process of entangled quantum objects. Slightly less than half of the answers described that the measurements of two entangled quanta are not independent and therefore correlated (category 2). The term "correlation" itself was rarely used in the responses. Instead, participants often invoked the concept of perfect correlation as exemplified by Bell states. This is illustrated by the following example:

> *"When two entangled particles are sent in different directions, and the spin of one of them is measured (e.g., spin up), the other particle automatically settles into a spin down state."* (Participant 20)

Seven subjects expressed ideas implying that an action is mediated between two entangled quantum objects (category 3). This often, but not always, referred to an action transferred during the measurement, as suggested by the following example quotation:

> *"The spin of one entangled quantum object is changed, so it is also changed in the other quantum object."* (Participant 29)

In this instance, the participant elucidated that altering one quantum object brings about a corresponding change in its entangled counterpart, irrespective of any measurement. Therefore one can say, action is being transmitted through the bond of entanglement. This

Table 2 Conceptions of quantum entanglement with regards to entangled states. Categories were inductively formed using answers from Q1 and Q2. N=31, Cohen's κ = 0.85, 95% CI: [0.72; 0.98]

Category	Coding rule	Anchoring example	N
1. Property of the system as a whole	The answer implies that with entangled quantum objects, only the system as a whole can be described. There is a common state of the entire system.	"The individual particles cannot be assigned their own state, but only the entire system."	8
2. Measurement correlation	The answer implies that in the case of entangled quantum objects, the measurement on one quantum object provides information about the second/other quantum objects.	"For example, if you want to study a pair of entangled electrons: you can measure the spin of an electron. That can be anywhere. The other electron is spatially separated. Without measurement, you now know the spin of the second electron as well."	15
3. Action between quantum objects	The answer implies that in entangled quantum objects, the change in one quantum object causes instantaneous changes in the second quantum object(s).	"If, for example, one changes the spin of one of these entangled electrons or fixes it by a measurement, the state of the other electron is also fixed immediately (without time delay), regardless of distance from the first electron."	7
4. Undefined relation of quantum objects	The answer implies that entangled quantum objects are otherwise connected or dependent on each other. (If none of the two above category holds)	"An entangled quantum state exists when the states of at least two quantum particles depend on each other."	6
5. Superposition	The answer confuses entanglement with superposition.	"An entangled state is a superposition of several states. A system is therefore in a superposition of several states as long as it is not measured. As soon as it is measured, it settles on one state."	4
6. Mathematical formalism	The answer involves mathematical formalism.	"Example: Bell state: A state of form $\left\vert\phi^-_{AB}\right\rangle = \frac{1}{\sqrt{2}}(\vert 0\rangle_A \vert 0\rangle_B - \vert 1\rangle_A \vert 1\rangle_B)$ of a composite system consisting of two particles A and B cannot be written as a product state. The state is not separable, and one cannot assign a particular state to either A or B."	3

quote uncovered a misunderstanding of participants belonging to this category. Namely, they believed that through entanglement one can knowingly change one quantum object and induce a change in the second entangled object and thereby control it. This goes as far as having the possibility of changing states forth and back.

Six other participants described entangled states as two or more quantum objects interacting without any measurement correlation or action mediation (category 4). Connections were instead, *"result of an interaction"* (participant 12) *"dependence"* (participant 8) or an *"energetic connection"* (participant 9)—without further explanation provided by the participants. Another participant explained the connection between particles with a kind of *consciousness* of the particles:

> *"The other particle 'knows' which property the first particle has decided on, even if the properties are only determined during the measurement."* (Participant 20)

Four participants conflated entanglement with superposition, as categorized under category 5. Finally, although the question explicitly invited to bring a mathematical example, there were only three participants who explained entanglement mathematically (category 6).

Interestingly, the first three categories co-occurred in a single response on three occasions. An example for an answer that falls into all three categories was given by participant 32:

> *"One photon is linearly polarised and the other is perpendicularly polarised. If the polarisation of one photon changes, the polarisation of the entangled photon* $\underbrace{\textit{also changes.}}_{\textit{3. Action}}$ *The* $\overbrace{\textit{overall result is a state}}^{\textit{1. System as a whole}}$ *that is achieved through the coupling. This means, for example, that the state (polarisation) of one photon can be measured, which means that the* $\underbrace{\textit{state of the other photon is known."}}_{\textit{2. Measurement correlation}}$ (Participant 32)

5.1.2 Conceptions of quantum entanglement with regards to quantum teleportation

The third question of the study phase 1 questionnaire was used to investigate conceptions of quantum teleportation (question Q3, see Table 1). An overview of the conceptions found alongside coding rules, anchor examples and occurrence frequencies is given in the category system in Table 3.

Here, a total of eight study participants expressed the notion that quantum teleportation involves the transfer of a state from one quantum object to another (category 1). However, one participant inferred from the state transfer concept that faster-than-light information transmission might be possible:

> *"If you measure one entangled quantum particle, you immediately know the state of the other entangled particle. Einstein's hypothesis was that there are hidden variables that allow these particles to communicate. This was done by re-measuring Bell's inequalities. This is called quantum teleportation because the state is 'teleported.' This means that information can be transmitted faster than the speed of light."* (Participant 43)

A total of five participants shared the opinion that quantum teleportation means a transmission of information faster than the speed of light (category 2). Related to this was the most commonly found notion that quantum teleportation can convey action instantaneously—and thus faster than light (category 3).

The idea that quantum teleportation, in more general, enables fast information transfer (category 4) was highlighted by four participants. Seven participants stated that quantum teleportation is possible over long distances (category 5) and another two participants explained quantum teleportation with the property of quantum objects not being localizable (category 6).

Table 3 Conceptions of quantum entanglement with regards to quantum teleportation. Categories were inductively formed using answers from Q3. N=31, Cohen's κ = 0.70, 95% CI: [0.53; 0.87]

Category	Coding rule	Anchor example	N
1. Transmission of states	The answer states that quantum teleportation is the transfer of one state to another quantum object.	"Thus, by establishing a state, one could 'send over' targeted states through the collapse of the system."	8
2. Faster-than-light transmission of information	The answer states that in quantum teleportation, information is transmitted at faster-than-light speeds.	"It means that with the help of quantum entanglement, information could be transmitted faster than the speed of light."	5
3. Instantaneous action	The answer describes quantum teleportation—the instantaneous transmission of an action without time delay over any distance.	"In this context, teleportation means the instantaneous transport of information."	16
4. Fast information transmission	The answer states that in quantum teleportation, information is transmitted at great speed.	"Entangled particles can send information over long distances in a short time."	4
5. Long distances	The answer says that quantum teleportation involves sending things over long distances.	"Similarly, teleportation involves sending an object or energy over long distances."	7
6. Spacelessness of particles	The answer states that quantum teleportation is possible because particles occupy several positions in space.	"Teleportation means in the sense that a particle can vary from position (no fixed place in space, contradicts classical idea)."	2
7. Skipping space	The answer states that in quantum teleportation space is "skipped"or there is no local interaction.	"Teleportation is when something is transported to a new location without crossing the intervening space. In this context, I think it's about teleporting a quantum."	3
8. Classical teleportation	The answer states that quantum teleportation makes particles disappear in one place and reappear in another.	"Teleportation means the disappearance of a particle at one measurand and the reappearance of the particle at another measurand."	5
Other	The answer does not fit into any of the above categories.	"Information transfer over (relatively) long distances without the quanta being 'connected'."	3

The term *teleportation* is taken up by three participants who stated that properties relocate in quantum teleportation (category 7), similar to the image of classical teleportation seen in science fiction in which an object can jump over space (category 8).

Three participants gave answers not matching any of the above categories. Among those students, one stated that quantum teleportation does, in fact, not enable faster-than-light-communication:

> *"Teleportation here refers to the instantaneous determination of the second measurement result once the first measurement result has been obtained. It does not matter whether the entangled system has been split and measured at distant points in space. However, information is not really teleported at superluminal speed here, because for the second system to determine a measurement result, it had to be physically random*

Table 4 Conceptions of quantum entanglement with regards to applications. Categories were inductively formed using answers from Q4. N=30, Cohen's κ = 0.78, 95% CI: [0.63; 0.92]

Category	Coding rule	Anchor example	N
1. General Answer	The answer is the general assessment that initial technological potential is underestimated, independent of entanglement and quantum technologies.	"Well, that's what people used to say about a lot of things, that they had no application. And a few years or decades later, applications have been found that we don't want to do without today. Who knows what will come in the future?"	10
2. Speculation	The answer speculates on what could be possible and how. If necessary, approaches and potentials for the use of entanglement are described without naming a specific technology.	"Nevertheless, I think the study of artificially entangled quantum states with limited numbers of particles certainly has potential and could be worth exploring."	7
3. Counterexample	The answer gives one or more counter examples/technology(s) or field(s) of application. a. Quantum computer b. Quantum teleportation/cryptography/communication c. Other, e.g., quantum sensors	"This statement is wrong, because quantum entanglement is very much used in applications today. Entanglement occurs in every quantum mechanical measurement, so it is ubiquitous. Quantum entanglement also plays a central role in quantum computers, which use quantum mechanical states for their calculations."	16 a: 12 b: 10 c: 1
Other	The answer does not fit into any of the above categories.	"You can't teleport anything alive. Only individual parts. You would have to split a human being into quanta to teleport him."	3

> *beforehand which result the first system (and the second system) would output."* (Participant 30)

Finally, one participant offered an ad-hoc hypothesis suggesting a potential connection to the tunneling effect (Participant 27).

5.1.3 Conceptions of quantum entanglement with regards to applications

The final question in the phase 1 questionnaire presented a statement claiming that quantum entanglement has theoretical implications but no practical relevance with regards to applications. The participants were asked to evaluate and judge this statement (question Q4, see Table 1). An overview of the conceptions found alongside coding rules, anchor examples and occurrence frequencies is given in the category system in Table 4.

Ten responses were of a general nature, stating that the technological potential is fundamentally underestimated, but that it would simply not yet be possible to assess how great the potential of quantum entanglement is or can be. For example, the answers referred to the Nobel Prize awarded in 2022 (Participant 23) or to the history of physics:

> *"But already many forward thinkers in physics were ridiculed for their farsightedness and declared crazy, although their concepts were confirmed decades or even centuries later and found application in technology."* (Participant 20)

Seven participants stated that entanglement is or will be relevant and speculated on what can and will be possible, without naming a specific technical application. For example, the possibility of using quantum entanglement for communication was mentioned. However, this again fell back on the conception that entanglement conveys instantaneous information transfer:

> *"It may be possible to use this knowledge for communication. If two particles are entangled and separated by large distances, information can be transmitted instantaneously; changing the state of one particle changes the state of the other."* (Participant 7)

Just over half of the participants (N=16) were able to name specific technologies and thus refute the statement given in the questionnaire item Q4. It is noteworthy that the majority of them (75%) also named the quantum computer technology.

Finally, there were three participants whose statements did not fit into any of the above categories. These included the statements that quantum entanglement *"has no relevance yet"* (Participant 24), that quantum entanglement could make it possible to teleport a human being *"if it is divided into individual parts"* (Participant 19) and that it has already *"been possible to entangle tardigrades"* (Participant 33).

5.2 Study phase 2 results

5.2.1 Conceptions of quantum entanglement with regards to entangled states

Five main categories of learners' conceptions of entangled states emerged from the analysis of study phase 1 data (see Table 2). Based on the student responses, we created one to four statements associated with each of these categories and asked the study phase 2 participants to rate their agreement with the respective statement. Our results are presented in diverging stacked bar charts, where items in line with the scientific view are marked with ✓ and items not in line with the scientific view with ✗:

1. *Property of the system as a whole* (items 1.1 to 1.2, see Fig. 2).
2. *Measurement correlation* (items 2.1 to 2.3, see Fig. 3).
3. *Action between quantum objects* (items 3.1 to 3.4, see Fig. 4).
4. *Undefined relation of quantum objects* (items 4.1 to 4.4, see Fig. 5).
5. *Superposition* (item 5.1, see Fig. 6).

For a detailed breakdown of all items within these categories, we refer the reader to Table 7. Additionally, we provide the item correlations in Table 10 where correlations below 0.10 have been suppressed.

Table 5 offers insights into the percentage of participants who responded to each item either in alignment with or in contrast to the current scientific viewpoint. These results are described in detail in the following subsections.

Category 1: Property of the system as a whole For the first item which states that *entangled quantum objects only have one common state* (item 1.1), a tendency towards agreement was found (average degree of agreement at μ = 3.12, *SD* = 1.36).

The second item states that *in the case of entangled states, the entire system must be considered as a whole, regardless of how far apart the individual subsystems are from each other* (item 1.2), was agreed to by most participants (μ = 4.18, *SD* = 1.36). This suggests that this concept of entangled states seems to be present in an even larger proportion of people than we found in study phase 1 (8 out of 31 there).

Table 5 Percentage of Answers (not) in line with scientific point of view

	Not in line with the scientific view	Undecided	In line with the scientific view
1. Property of the system as a whole			
1.1	36%	18%	47%
1.2	14%	6%	81%
2. Measurement correlation			
2.1	77%	12%	11%
2.2	29%	15%	56%
2.3	60%	22%	18%
3. Action between quantum objects			
3.1	84%	6%	11%
3.2	38%	14%	48%
3.3	59%	16%	25%
3.4	33%	32%	36%
4. Undefined relation of quantum objects			
4.1	71%	11%	18%
4.2	41%	25%	34%
4.3	49%	18%	33%
4.4	11%	40%	49%
5. Superposition			
5.1	23%	37%	40%

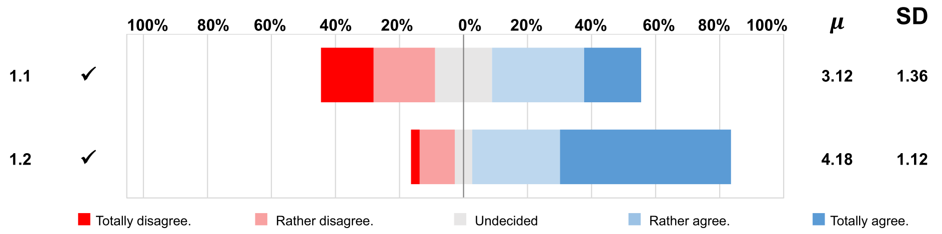

Figure 2 Diverging Stacked Bar Chart of items belonging to the category *Property of the system as a whole*

Category 2: Measurement correlation The idea that entanglement was explained as a concept of measurement correlations, as described by this category, was voiced by the highest percentage of participants in the first phase of the study. A similar trend was observed in the agreement with related items in the second phase of the study, where participants consistently agreed with all the items.

For items 2.1 and 2.3 similar response patterns are apparent (Spearman's $\rho = 0.73$, $p < 0.001$) with similar mean ratings (item 2.1: $\mu = 4.18$ $SD = 1.18$; item 2.3: $\mu = 3.74$, $SD = 1.36$). One possible explanation for this might have been the similar wording of the items. Nevertheless, these two items are rather "not in line with the scientific view", as they imply that quantum entanglement always coincides with perfect (anti-)correlation.

The second item in this category (item 2.2), which mitigates this very property by stating that *only probability statements follow from the measurement of entangled quantum objects* was surprisingly rejected by more people than the other two items.

Category 3: Action between quantum objects In study phase 1, seven out of 31 participants expressed the idea that quantum entanglement could mediate action. This was manifested, for example, by a change in one quantum object causing a change in another quantum object entangled with it. More participants agreed than disagreed with this last idea,

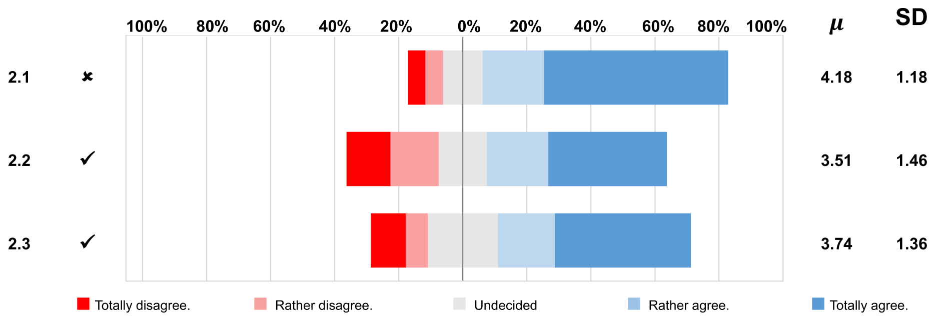

Figure 3 Diverging Stacked Bar Chart of items belonging to the category *Measurment correlation*

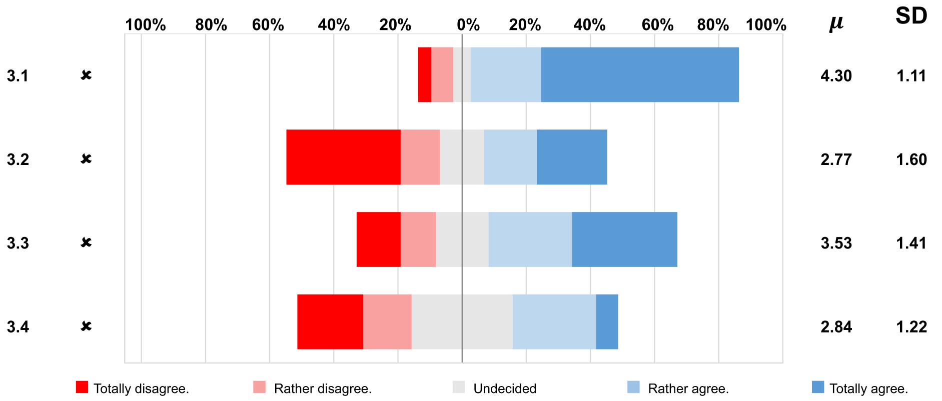

Figure 4 Diverging Stacked Bar Chart of items belonging to the category *Action between quantum objects*

as evidenced by a mean of $\mu = 3.53$ ($SD = 1.41$) on item 3.3. Thus, study phase 2 was able to validate the concept found in study phase 1.

Item 3.1, which states that *entangled quantum objects influence each other*, was represented with an even higher agreement ($\mu = 4.30$, $SD = 1.11$) among the participants. This could be explained by the wording of this item. For example, some participants might have understood "influence" to mean "depend", as the participants' comments showed. Nevertheless, this high level of agreement indicates a high prevalence of this category of concept.

Some participants derived a possible consequences of this influence in study phase 1 which were represented by items 3.2 and 3.4 (themselves correlated by Spearman's $\rho = 0.69$, $p < 0.001$). Namely, *this mediation of action can enable faster-than-light* (item 3.2) or *instantaneous communication* (item 3.4). Participant 7, to name an example, showed this conception (illustrated by a quotation that can be found in Sect. 4). These items though were agreed with less which is illustrated by their lower means of $\mu = 2.77$ ($SD = 1.60$) and $\mu = 2.84$ ($SD = 1.22$).

Category 4: Undefined relation of quantum objects This category includes all notions that refer to entangled quantum objects being in some kind of relation to each other, that have nothing to do with measurement correlation, and that do not describe the mediation of action.

The first two items examined participants' ideas about the preparation of entangled quantum states: Item 4.1 referred to a metaphor echoing a similar statement of a par-

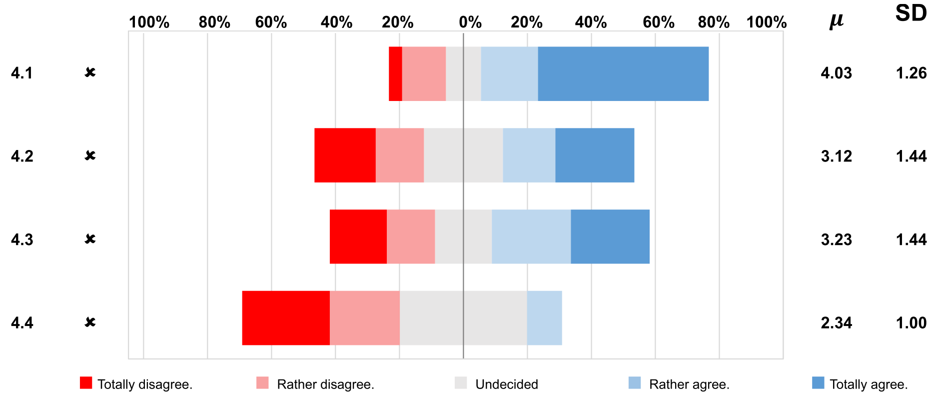

Figure 5 Diverging Stacked Bar Chart of items belonging to the category *Undefined relation of quantum objects*

ticipant in study phase 1: *The "connection" in entangled states is like a postcard that is torn in half and then sent in two envelopes. When you open one envelope, it is immediately clear over long distances what is enclosed in the other* (item 4.1). This item made the claim that even before measuring one entangled quantum objects, similar to the classical picture of envelopes, it is actually predetermined how the measurement will turn out, a claim that is generally differing from the scientific view as it claims that local hidden variables explain the outcome of the measurement of entangled states.

The second item 4.2 has a similar statement, describing this fact without a classical metaphor. The majority of study participants agreed with both items 4.1 and 4.2, with a larger average for the metaphor (item 4.1: $\mu = 4.03$, $SD = 1.26$; item 4.2: $\mu = 3.12$, $SD = 1.44$).

Item 4.3 states that *the entangled quantum objects are in such a connection that one quantum object knows how a second quantum object behaves or is measured and accordingly it adapts itself knowingly*. Hence, this item ascribes a form of consciousness to quantum objects. The majority agreed with this item, as indicated by an average rating of $\mu = 3.23$ ($SD = 1.44$).

The statement that *spin entanglement also causes correlations in measurements of other observable* (item 4.4), which is not in line with the scientific view, was (correctly) rejected by the majority of participants ($\mu = 2.34$, $SD = 1.00$).

Category 5: Superposition Lastly, the confusion of entanglement and superposition—apparent of a significant amount of learners in study phase 1—was probed with item 5.1. A percentage of 23% (rather) agreed with this statement, showing that this conception found in study phase 1 could be replicated (average rating $\mu = 2.77$, $SD = 1.21$).

5.2.2 Conceptions of quantum entanglement with regards to quantum teleportation

An overview of the answer distribution as well as the mean ratings and standard deviations for each item with regards to quantum teleportation can be found in Fig. 7. Our results are presented in diverging stacked bar charts, where items in line with the scientific view are marked with ✓ and items not in line with the scientific view with ✗.

A detailed list of all items can be found in Table 8. In addition, for all items the percentage of participants who answered the corresponding item in line or not in line with

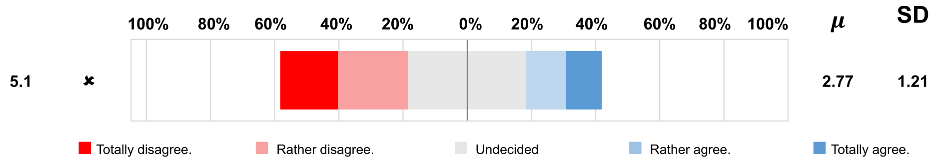

Figure 6 Diverging Stacked Bar Chart of items belonging to the category *Superposition*

Table 6 Percentage of Answers (not) in line with scientific point of view

	Not in line with the scientific view	Undecided	In line with the scientific view
Teleportation			
T1	19%	44%	37%
T2	44%	40%	16%
T3	41%	37%	22%
T4	58%	36%	7%
T5	58%	29%	14%
T6	21%	53%	26%
T7	8%	33%	59%
T8	10%	37%	53%

the scientific view is given by Table 6. Table 11 provides an overview of all Spearman's ρ coefficents as an indicator of item correlation. All values $|\rho| \leq 0.10$ are suppressed.

It is noteworthy that with regards to the items on quantum teleportation, a great number of participants was undecided when answering the items (overall mean percentage of *undecided* votes across all items: 39%).

Item T1 states that *quantum teleportation corresponds to the transfer of a quantum object's state to another quantum object*, yielding an average rating of $\mu = 3.25$ ($SD = 1.10$). Remarkably, despite its alignment with the scientific view, this particular item garnered less unanimous agreement than the four following items.

Upon initial inspection, a shared response pattern was discernible across items T2 (*instantaneous transfer of an action to quantum objects*), T3 (*faster-than-light information transfer*), T4 (*non-local interaction*), and T5 (*spatially separated quantum objects exchange information*). Likewise, the average ratings and standard deviations for these items were close to equivalent: T2 ($\mu = 3.36$, $SD = 1.18$), T3 ($\mu = 3.19$, $SD = 1.30$), T4 ($\mu = 3.74$, $SD = 1.03$), and T5 ($\mu = 3.64$, $SD = 1.12$). In particular, all mean ratings imply a tendency to agreement with the respective statements.

Additionally, an examination of the correlations among these four items revealed a strong positive relationship between Spearman's $\rho = 0.33$ and $\rho = 0.60$ ($p < 0.01$, for details see Table 11). The strong correlations among these four statements, which are not in line with the scientific view, indicated a fundamental conception related to all items.

Item T6 states that *quantum teleportation is made possible by the innate capability of quantum objects to exist in multiple positions simultaneously, thereby evading strict localization*. This statement garnered an average rating of $\mu = 2.82$ ($SD = 1.07$). This paralleled the findings of study phase 1, wherein a mere two participants fell within the corresponding classification (see category 6 in Table 3).

Item T7, suggesting that *quantum teleportation entails the physical displacement of a quantum object between locations*, received an average rating of $\mu = 2.15$ ($SD = 1.04$). This signaled discord with the statement, mirroring outcomes from study phase 1, wherein only

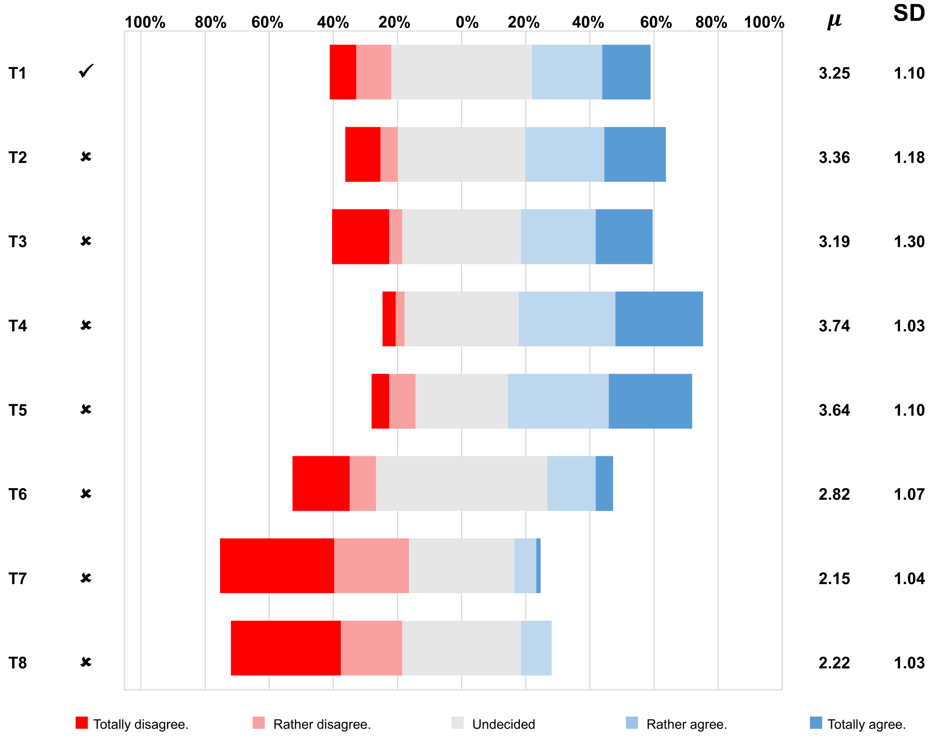

Figure 7 Diverging Stacked Bar Chart of items with regards to *Quantum teleportation*

three participants expressed the conception of quantum teleportation involving spatial leaps (category 7) or likened it to conventional teleportation, an opinion expressed by five participants (category 8).

Lastly, the eighth statement T8, which draws a connection between quantum teleportation and the tunneling effect, garnered a notably low average rating of $\mu = 2.22$ ($SD = 1.03$). Consequently, it can be reasonably concluded that this idea differing from the scientific view is held only by a limited number of participants, similarly aligned with the lone participant reflecting this conception in study phase 1.

5.2.3 *Conceptions of quantum entanglement with regards to applications*

An overview of the answer pattern as well as the mean and standard deviation of the ratings of each item with regards to applications can be found in Fig. 8. A detailed list of all items can be found in Table 9. Table 12 provides an overview of all Spearman's ρ coefficents as an indicator of item correlation. All values $|\rho| \leq 0.10$ are suppressed.

Item A1, which states that *all physical findings find an application at some point and thus become technically relevant in the long term*, was (rather) agreed by a share of 41% of the study participants compared to a share of 36% who (rather) disagreed with this opinion. Interestingly, this general point of view was not found to be statistical significantly correlated with any other item belonging to this domain.

Both items A2 and A3 state that *quantum entanglement is already of relevant technical importance* (item A2) or *will become so in the future* (item A3). The average ratings (item A2: $\mu = 3.49$, $SD = 1.13$; item A3: $\mu = 3.63$, $SD = 1.15$) indicate that the majority of the participants were convinced of this.

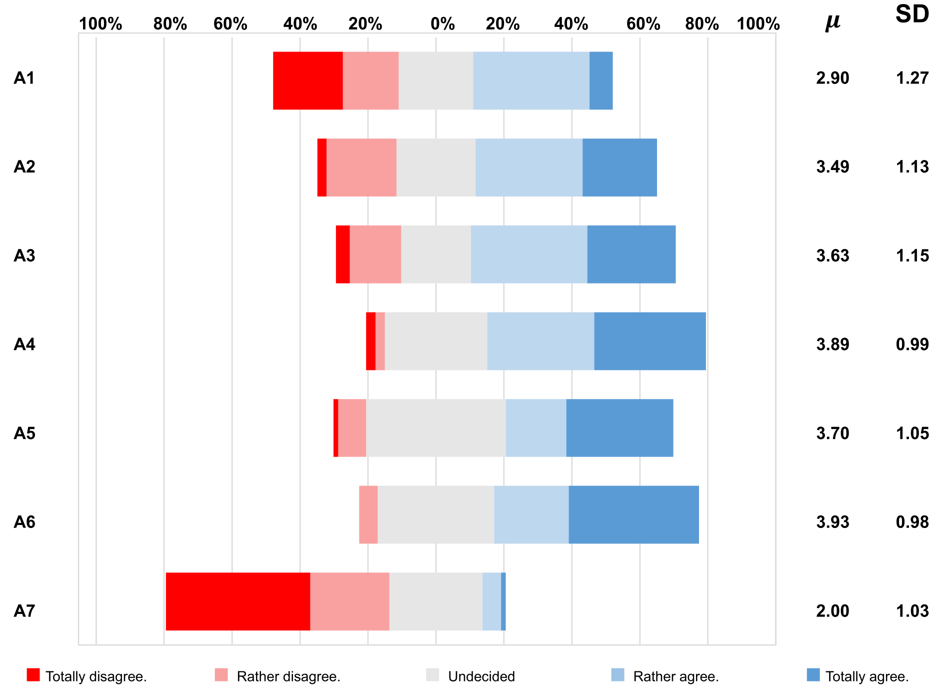

Figure 8 Diverging Stacked Bar Chart of items with regards to *Application of Quantum Entanglement*

High overall agreement was obtained for items stating that quantum entanglement is applied in the following techniques: *quantum cryptography* (item A4: μ = 3.89, *SD* = 0.99), *tap-proof key distribution* (item A5: μ = 3.70, *SD* = 1.05) and *quantum computing* (item A6: μ = 3.93, *SD* = 0.98). Each of these applications, in fact, utilizes quantum entanglement in some of their approaches, although it's important to highlight that not all quantum cryptography or key distribution methods necessarily rely on entangled states, as demonstrated by the BB84 protocol, for instance.. Also noticeable was the high correlation of these three items (Spearman's $\rho \in [0.48; 0.64]$, $p < 0.001$), which points towards the fact that if someone is familiar with one technology, they are likely to have some knowledge of other technologies related to the applications of quantum entanglement. One could also identify a correlation between these three items and item A2 (*quantum entanglement is already of technical importance*), which could be expected as someone that was aware of technical applications would also notice a relevance of this technology.

That *it will someday be possible to teleport physical objects and eventually even people* was denied by a majority of the participants, which was expressed in low agreement with item A7 (μ = 2.00, *SD* = 1.03).

6 Discussion

This study explored what views of quantum entanglement are widespread among learners and specifically among pre-service physics teachers. Frequently found ideas that are not in line with the scientific views are summarized and discussed in the following (and referred to as **C1** to **C6**).

6.1 Conceptions of quantum entanglement with regards to entangled states

We found that learners widely believe that one must always consider the entire system when dealing with entangled states: Entangled quantum objects were described as having a common state, irrespective of the spatial distance between individual subsystems. This conception is in line with the scientific view. Likewise, the following conception is also in line with the scientific view: The measurement of one quantum object enables the possibility of making probability statements about the same measurement of another quantum object entangled with it because the measurements are correlated. The majority of study participants also agreed with this conception (item 2.2).

Even more participants though agreed with the view that the measurement not only makes probability statements possible, but implies certain conclusions (items 2.1 and 2.3). However, this idea is generally not correct in such a way, but only if the entangled quantum objects are in a maximally entangled state, as it is for example the case with the Bell states. Consequently, the following conception not in line with the scientific view could be frequently identified:

C1 *Measurements with entangled states always show perfect (anti-)correlations.*

This finding is consistent with the results obtained by Konhle et al. [51], who also recognised this as a widely held misconception in the context of quantum entanglement.

Another conception that differs from the scientific view, which was frequently identified in study phase 1, could also be confirmed through items 4.1 and 4.2 of study phase 2. The following conception is expressed:

C2 *The outcome of the measurement of quantum states in entangled states can be explained through local hidden variables.*

If someone believes that local hidden variables are responsible for clarifying the outcomes of measurements involving entangled quantum objects, they adhere to the idea that certain underlying, unobservable factors decide the results of these measurements. According to this idea, the entanglement between quantum objects does not directly determine their behavior; instead, these local hidden variables are thought to govern the observed correlations between the entangled particles *before* they separate from each other. To exemplify, this notion is illustrated by the envelope metaphor used in item 4.1. As these are classical images the result of measurement is predetermined, with someone deciding beforehand over the result of the measurement by putting the postcard in either envelope.

Similarly EPR (short for Einstein, Podolsky and Rosen) [56] famously assumed this to be case for entangled quantum objects. Nevertheless this idea has been proven to be wrong [57, 58]—partly the reason, why the 2022 Nobel price has been awarded.

This perspective implies that learners seek for a classical explanation of the non-locality of quantum entanglement. This effect can also be seen in research from Aehle et al. [59], wherein students also resort to the classical image of hidden variables explaining the result of measurement on entangled quantum objects. As observed in earlier research into learners' conceptions the fallback to classical explanations is a typical obstacle when learning quantum mechanics [23].

In the first study phase, analysing the responses revealed the idea that entanglement enables the mediation of action. Therefore, if an action (such as change or measurement) is performed on one quantum object, its entangled counterpart will also undergo an action. This fact diverges from the scientific view. Still, seven out of the 31 participants in study phase 1 belonged to the category encapsulating this conception. In study phase 2 this conception was probed by items 3.1 and 3.3 with which a majority of participants also agreed. Hence, one can commonly find the following conception:

C3 *Quantum objects in entangled states can influence each other.*

Some participants explained conception C3 during study phase 1 using wording that suggested quantum objects have a consciousness of each other. This perspective suggests that quantum objects not only exhibit entanglement but also have a form of consciousness or awareness that enables them to interact and communicate in ways that transcend our current understanding of physics. As a result, it was assumed that this is a notion held by some of the learners. This fact could also be validated by the majority of participants agreeing with item 4.3. Therefore, we found the following conception not in line with the scientific view:

C4 *Quantum objects in an entangled state have some kind of consciousness of each other.*

Finally, the confusion between entanglement and superposition, already noted by Ref. [36, 51] was observed among a small percentage of participants (10% in study phase 1 and 23% in study phase 2):

C5 *The entanglement of states is the same as the superposition of states.*

Belonging to C5 it should be noted that superposition and entanglement are not unrelated to each other, as superposition is a mandate for entanglement. Yet, they cannot be used as synonymous.

6.2 Conceptions of quantum entanglement with regards to quantum teleportation

Firstly, in study phase 2 it was observed that an overall mean of 39% selected the option "Undecided" for items related to quantum teleportation. This could suggest that only a few students are at ease answering questions regarding quantum teleportation which might be explained by the lack of awareness regarding this subject. This highlights the fact that learning about quantum teleportation is not widely practiced. One possibility to deal with the topic of quantum teleportation in class and to "experience" entanglement in an experiment is offered in Ref. [60]. Furthermore, to put quantum teleportation in a teaching context, the topic can be used to introduce students into quantum information theory in secondary school [61].

Nevertheless, most of the participants held a concept consistent with the scientific perspective that quantum teleportation involves the transfer of a quantum object's state to another object (item T1). This contrasted with the notion depicted in science fiction that a quantum object skips physical space and reappears at another location during quantum teleportation (item T7). However, more respondents disagreed than agreed with the latter idea.

The four items T2, T3, T4, and T5, which are related to the categories 2 to 5 found in study phase 1 (see tabel 3), were found to have strong correlations with each other. The underlying students' idea is particularly interesting as all four items express beliefs not aligned with the scientific view. This idea can be summarized as follows:

C6 *Quantum entanglement enables faster-than-light communication.*

An illustration of how this conception relates to the four items T2 to T5 is given by an answer to question 3 in the first questionnaire. The quote includes statements similar to the items themselves:

> “ $\overbrace{\textit{Particle X at location A interacts with particle Y at location B.}}^{\textit{T4: Non-local interaction}}$ *If both particles are in a common state, I can* $\underbrace{\textit{project all information}}_{\textit{T5: Exchange of information}}$ *from place B to place A (and vice versa)* $\overbrace{\textit{without time shift}}^{\textit{T3: Faster-than-light}}$ *and over large distances. Thus I can* $\underbrace{\textit{map the particle X at place A also immediately at place B.}}_{\textit{T2: Transfer of action}}$” (Participant 12)

While quantum entanglement involves instantaneous correlations between particles, these correlations cannot be used to transmit information faster than the speed of light, as explained in Sect. 2.1.

The conception that quantum entanglement enables faster-than-light transfer of information has also been noticed in Ref. [26]. Further, it is noticed that this perception could even “do enormous harm” ([26], p.32). Students might misinterpret this idea to be used in *explaining* “faith healing”. The author suggests using an analogy to electrodynamics when encountering this belief.

6.3 Conceptions of quantum entanglement with regards to application

Regarding the technological relevance, most participants recognised the importance of entanglement—now or in the future—for quantum technologies such as quantum cryptography/key distribution (and therefore quantum communication), and quantum computing. While in study phase 1 quantum computing was mentioned remarkably often, in study phase 2 there was no higher agreement for the item of quantum computing visible than for cryptography.

This parallels the finding of Ref. [62], where expert surveys were conducted to derive predictions on the future relevance of quantum technologies. The experts recognized quantum sensing and quantum communication to already be important today and tended to quantum computing to be most the important quantum technology in the future.

Following the noticed technological relevance of entanglement, this supports the need for a good education on entanglement to prepare for the future needs of e.g. engineers who have to work with these technologies that use quantum effects like entanglement. Likewise, educators should undergo appropriate training to effectively facilitate the learning of quantum technologies within educational institutions.

In study phase 1, 10 subjects (about 33%) expressed the opinion that the technological potential of scientific findings is typically underestimated, which is why it can be assumed that quantum entanglement can also make a technological contribution. Item A1 has shown the divided opinion on the general technical relevance of physical findings in study phase 2. While 41% agreed and might therefore argue that also entanglement will have technical relevance, almost the same number of participants disagreed. Similar results have been expected as the participants of the study consisted only of physics students. A general optimism regarding the relevance of certain topics could therefore be assumed. This can be explained by the fact that students learn about similar historical occasions in their academic education, where technological relevance has been underestimated, e.g., the early disbelief in quantum physics. They therefore tend to transfer this experience to the topic of entanglement. Since disagreement with the other items on the technical relevance of entanglement was smaller, at least some participants have really seen the technical relevance of entanglement.

7 Implications

Our findings allow to draw implications for both, educational research and practice a few of which we touch upon in the following subsections.

7.1 Implications for educational research

The analysis of the collected data indicates that pre-service physics teachers often possess incomplete or misconceived notions about quantum entanglement. Consequently, it is imperative to gain deeper insights into the prevalence of these conceptions. The examination of the reasons for individual attitudes and misunderstandings needs to be a key element for future research as this is the only way to fully understand them, which in turn enables the development of evidence-based teaching concepts.

One example of a study design addressing this research desideratum with regards to a topic within quantum physics can be found in Ref. [63]. There, the authors present a typology of learning impediments which they use to explain students' misunderstandings of potential wells and tunneling.

Further, it should be noted that our study only gives an insight into learners' conceptions of entanglement at a certain time probed with closed items. We did not investigate (a) conceptual understanding and (b) development of the conceptions at hand. An examination of persisting conceptions that are not in line with the scientific view following a pedagogical intervention could offer intriguing insights. Future research could for instance use our findings on conceptions to develop a quasi experimental intervention study. Similarly, Bitzenbauer [64] has previously investigated the effect of a teaching concept on preuniversity students' conceptions about quantum physics.

Given that the topic of entanglement is primarily explored at the university level, it becomes relevant to delve beyond qualitative concepts. It is essential to investigate the impact of a mathematical representation of entangled systems on conceptual comprehension. There is still a scientific debate regarding whether relying solely on mathematical formalism is helpful in addressing conceptual or interpretational challenges. Simultaneously, a mathematical description can assist in the development of a functional understanding. Therefore, the question of how the early and systematic introduction of entanglement in the sense of formalism helps to prevent widespread misconceptions and promote conceptual development is of considerable importance.

This aspect, which was not addressed in this study, has been partially explored in other studies, such as [65]. Another relevant study previously investigated the introduction of mathematical formalism to describe quantum mechanics in secondary education [66], and its findings could serve as a valuable reference.

7.2 Implications for educational practice

Our data analysis has unveiled a prevalent tendency among numerous students to conflate the principles of entanglement and superposition. While it is not assured that all participants in our study have received formal instruction on quantum entanglement, and thus this conflation may not necessarily result from shortcomings in teaching, it does underscore the importance of addressing this misunderstanding through pedagogical methods. Therefore, educators should consider implementing a clear differentiation between these two concepts in their teaching practices. Achieving this objective could involve the incorporation of suitable illustrations from popular science, such as the renowned paradox of

Erwin Schrödinger's cat [67]. In this scenario, Schrödinger's cat, alongside the radioactive atom, serves to elucidate the concept of superposition, wherein the cat exists in a quantum state concurrently as both alive and dead, while the radioactive atom simultaneously occupies states of decay and non-decay until subjected to observation. In contrast, entanglement depicts the inseparability between the cat's and radioactive particle's state.

As shown by other studies [36, 68], utilizing games is practicable and advantageous in comprehending entanglement and superposition. So-called "games with a purpose" are useful in the context of quantum physics concepts, especially because they allow us to "explore and experience counter-intuitive quantum behavior" ([36], p.3). In Ref. [68], learners showed a superior understanding of the concept of entanglement among learners who played games compared to the control group.

The conception, not in line with the scientific view, that measurements on entangled quantum objects always show perfect (anti-)correlation, could also commonly be found among learners. Yet, this is clearly a problem that can be addressed through teaching practice. Typically the most prominent Bell states are being used as an example to introduce and explain quantum entanglement. Problematically, these only show perfect correlation, which hinders students to learn at all about non-perfect correlation. The latter is the observation one can make when dealing with non-maximal entanglement. This can and should be addressed by giving more examples or using a simulation which includes non-perfect correlations as well, e.g., $|\psi\rangle = \frac{1}{\sqrt{3}}(|10\rangle + |01\rangle + |00\rangle)$ [65].

8 Conclusion

Based on the data we gathered and analyzed during study phase 1, we successfully identified a range of concepts apparent among pre-service physics teachers and undergraduate physics students pertaining to quantum entanglement across three domains: entangled quantum states, quantum teleportation, and applications of quantum entanglement. Subsequently, these concepts were further validated in a second phase of the study.

Concerning entangled states, a notable alignment with the scientific viewpoint was observed among the respondents. They recognized that when dealing with entangled quantum objects, it is imperative to consider the state of the entire system. Furthermore, a consensus emerged among most participants that the measurement of entangled quantum objects leads to (anti-)correlated outcomes. Nevertheless, regarding entangled states, five students' conceptions were identified that deviate from the scientific consensus (see Sect. 6.1).

The responses obtained during study phase 1, coupled with the hesitancy observed in study phase 2, unveiled a prevalent pattern among learners. Many of them possess only a limited number of ideas or, in some cases, conceptions that run divergent to the established scientific understanding of quantum teleportation. While some learners correctly articulated the concept that quantum teleportation entails the transfer of a state from one quantum object to another, aligning with the scientific perspective, a significant portion of learners held the conception that quantum entanglement enables faster-than-light communication via quantum teleportation (see Sect. 6.2).

Regarding applications of quantum entanglement, our findings indicate that a part of our study participants demonstrate an awareness of the significance of quantum entanglement in contemporary technologies, including quantum computing and quantum cryptography/key distribution (see Sect. 6.3).

Appendix A: Items of study phase 2

Table 7 Rating scale items included in the study phase 2 questionnaire, focusing on aspects of entangled quantum states, developed based on the categories that emerged from the study phase 1 data. For each item, it is indicated whether the item is not in line or in line with scientific views

Item Description	Not in line with the scientific view	In line with the scientific view
Property of the system as a whole		
1.1 A system of several entangled quantum objects has only one common state		x
1.2 Quantum objects in an entangled state must be considered as a total system, regardless of the spatial separation of the subsystems.		x
Measurement correlation		
2.1 Two quantum objects A and B are in a state that is entangled with respect to spin. If one measures the spin of A, the spin state of B is thereby determined.	x	
2.2 Two quantum objects A and B are in a state that is entangled with respect to spin. A spin measurement on A leads to probability statements about the same measurement on B.		x
2.3 Two quantum objects A and B are in a state that is entangled with respect to spin. If one measures the spin of A, one knows the measurement result of the same measurement on B.	x	
Action between quantum objects		
3.1 Quantum objects in an entangled state can influence each other over arbitrarily large distances.	x	
3.2 Since entangled quantum objects influence each other across spatial separation, faster-than-light information transmission is possible.	x	
3.3 Two quantum objects A and B are in a state that is entangled with respect to spin. The change in the spin of A causes a change in the spin of B.	x	
3.4 Entanglement allows information to be transmitted instantaneously and can therefore also be used for faster data transmission.	x	
Undetermined relation of quantum objects		
4.1 The "connection"in entangled states is like a postcard that is torn in half and then sent in two envelopes. If you open one envelope, it is immediately clear over long distances what is in the other.	x	
4.2 When measuring entangled quantum objects, there is no interaction between the objects. Only one of the properties already determined at the time of creation is queried.	x	
4.3 A quantum object knows how another quantum object entangled with it behaves/is measured and takes on properties accordingly.	x	
4.4 Two quantum objects A and B are in a state that is entangled with respect to spin. The measurement on A and B with respect to an observable (other than spin) is also correlated.	x	
Superposition		
5.1 Entanglement is the superposition of states.	x	

Table 8 Rating scale items included in the study phase 2 questionnaire, focusing on quantum teleportation, developed based on the categories that emerged from the study phase 1 data. For each item, it is indicated whether the item is not in line or in line with scientific views

Item Description	Not in line with the scientific view	In line with the scientific view
Teleportation		
T1 In quantum teleportation, a state of one quantum object is transferred to another.		x
T2 Quantum teleportation describes the instantaneous transfer of an action to quantum objects over any distance.	x	
T3 Quantum teleportation describes faster-than-light information transfer for quantum objects.	x	
T4 Quantum teleportation describes a non-local interaction of quantum objects.	x	
T5 Quantum teleportation causes spatially separated quantum objects to exchange information.	x	
T6 Quantum teleportation is possible because quantum objects can assume several positions in space simultaneously and are therefore not localised.	x	
T7 Quantum teleportation involves transporting a quantum object from one place to another.	x	
T8 Quantum teleportation describes the tunnel effect.	x	

Table 9 Rating scale items included in the study phase 2 questionnaire, focusing on applications of quantum entanglement, developed based on the categories that emerged from the study phase 1 data

Application
A1 All physical findings find an application at some point and thus become technically relevant in the long term.
A2 Entanglement currently has technical relevance.
A3 Entanglement is not yet technically relevant but will be in the future.
A4 Entanglement finds application in quantum cryptography.
A5 By means of interleaving, a physically tap-proof key transmission can be realised, whereby data can be securely encrypted for a data distribution.
A6 Entanglement finds application in quantum computing.
A7 By using quantum physical entanglement, it might someday be possible to teleport physical objects and perhaps even people.

Appendix B: Item correlations between the items of the second questionnaire

Table 10 Domain (i): Spearman's ρ as a measure of the correlation between participants' rating $|\rho| < 0.20$ weak correlation, $0.20 \leq |\rho| \leq 0.30$ medium correlation, $|\rho| > 0.30$ strong correlation

	1.1	1.2	2.1	2.2	2.3	3.1	3.2	3.3	3.4	4.1	4.2	4.3	4.4	5.1
1.1	–													
1.2	0.12	–												
2.1	0.16	0.48	–											
2.2		0.15		–										
2.3	0.22	0.42	0.73		–									
3.1	–0.13	0.21	0.29	0.13	0.41	–								
3.2	–0.16	–0.17	0.25	–0.15	0.28	0.69	–							
3.3		0.11	0.23		0.12	0.21	0.15	–						
3.4		–0.29	–0.13	–0.24	–0.22	0.18	0.69	0.21	–					
4.1		0.19	0.20	0.29	0.15	0.30	0.15	0.17	–0.17	–				
4.2	0.40		0.13	0.26	0.11	–0.16	–0.12		–0.16	0.27	–			
4.3	–0.13		0.17	–0.13	0.33	0.22	0.23	0.12	0.14	–0.15		–		
4.4		–0.27	–0.21	–0.13	–0.20	0.26		0.23	0.12	–0.23	0.12	–0.16	–	
5.1	0.20	–0.12				–0.21		0.12		–0.23	0.12	–0.16	0.12	–

Table 11 Domain (ii): Spearman's ρ as a measure of the correlation between participants' rating $|\rho| < 0.20$ weak correlation, $0.20 \leq |\rho| \leq 0.30$ medium correlation, $|\rho| > 0.30$ strong correlation

	*T*1	*T*2	*T*3	*T*4	*T*5	T6	T7	T8
T1	–							
T2	–0.18	–						
T3	–0.18	0.60	–					
T4		0.40	0.33	–				
T5		0.22	0.44	0.43	–			
T6	–0.12			0.13	–0.28	–		
T7		–0.31	–0.27	–0.54	–0.41		–	
T8	–0.19	0.22	0.23	–0.13	–0.26	0.13	0.22	–

Table 12 Domain (iii): Spearman's ρ as a measure of the correlation between partcipants' rating $|\rho| < 0.20$ weak correlation, $0.20 \leq |\rho| \leq 0.30$ medium correlation, $|\rho| > 0.30$ strong correlation

	A1	*A*2	A3	*A*4	A5	*A*6	A7
A1	–						
A2		–					
A3		–0.11	–				
A4		0.38		–			
A5		0.39	0.21	0.64	–		
A6		0.40	0.17	0.61	0.48	–	
A7		–0.26		–0.33		–0.18	–

Acknowledgements
We thank Kim-Alessandro Weber for valuable contributions to this study and Stefan Heusler (University of Münster) for providing us with items on quantum entanglement.

Author contributions
M.B. and P.B. designed the study, M.B., F.G., M.S.U. and P.B. developed the questionnaires and collected the data, M.B., F.G. and P.B. analyzed the data, M.B., F.G., F.H. and P.B. wrote the first draft of the manuscript, M.S.U and H.F. reviewed the manuscript and edited the manuscript. P.B. supervised the study. All authors have read and agreed to the submitted version of the manuscript.

Funding
Open Access funding enabled and organized by Projekt DEAL. No funding was received for conducting this study.

Data availability
The data presented in this study are available on request from the corresponding author.

Declarations

Financial Interests
The authors have no relevant financial or non-financial interests to disclose.

Ethics approval and consent to participate
Ethical review and approval was not required for the study on human participants in accordance with the local legislation and institutional requirements.

Consent for publication
The participants provided their written informed consent to participate in this study.

Competing interests
The authors declare no competing interests.

Author details
[1]Institut für Didaktik der Physik, Universität Leipzig, Prager Straße 36, Leipzig, 04317, Germany. [2]Institut für Fachdidaktik der Naturwissenschaften, Technische Universität Braunschweig, Bienroder Weg 82, Braunschweig, 38106, Germany. [3]Didaktik der Physik, Department Physik, Friedrich-Alexander-Universität Erlangen-Nürnberg, Staudstr. 7, Erlangen, 91058, Germany.

Received: 16 September 2023 Accepted: 28 April 2024 Published online: 10 May 2024

References
1. Wang K, Song Z, Zhao X, Wang Z, Wang X. Detecting and quantifying entanglement on near-term quantum devices. npj Quantum Inf. 2022;8(1):1–11. https://doi.org/10.1038/s41534-022-00556-w.
2. Horodecki R, Horodecki P, Horodecki M, Horodecki K. Quantum entanglement. Rev Mod Phys. 2009;81(2):865–942. https://doi.org/10.1103/RevModPhys.81.865.
3. Jozsa R. Entanglement and Quantum Computation. 1997. https://doi.org/10.48550/arXiv.quant-ph/9707034.
4. Nadlinger DP, Drmota P, Nichol BC, Araneda G, Main D, Srinivas R, Lucas DM, Ballance CJ, Ivanov K, Tan EY-Z, Sekatski P, Urbanke RL, Renner R, Sangouard N, Bancal J-D. Experimental quantum key distribution certified by Bell's theorem. Nature. 2022;607(7920):682–6. https://doi.org/10.1038/s41586-022-04941-5.
5. Zhang W, van Leent T, Redeker K, Garthoff R, Schwonnek R, Fertig F, Eppelt S, Rosenfeld W, Scarani V, Lim CCW, Weinfurter H. A device-independent quantum key distribution system for distant users. Nature. 2022;607(7920):687–91. https://doi.org/10.1038/s41586-022-04891-y.
6. The Nobel Committee for Physics: Scientific Background on the Nobel Prize in Physics 2022. The Royal Swedish Academy of Sciences; 2022.
7. Venegas-Gomez A. The quantum ecosystem and its future workforce: a journey through the funding, the hype, the opportunities, and the risks related to the emerging field of quantum technologies. PhotonicsViews. 2020;17(6):34–8. https://doi.org/10.1002/phvs.202000044.
8. Hughes C, Finke D, German D-A, Merzbacher C, Vora PM, Lewandowski HJ. Assessing the needs of the quantum industry. IEEE Trans Ed. 2022;65(4):592–601. https://doi.org/10.1109/TE.2022.3153841.
9. Greinert F, Müller R, Bitzenbauer P, Ubben MS, Weber K-A. Future quantum workforce: competences, requirements, and forecasts. Phys Rev Phys Educ Res. 2023;19(1):010137. https://doi.org/10.1103/PhysRevPhysEducRes.19.010137.
10. QUCATS: Quantum Flagship: The Future Is Quantum. 2023. https://qt.eu/.
11. Greinert F, Müller R. European competence framework for quantum technologies. 2023. https://doi.org/10.5281/zenodo.7827254.
12. Sherson J, Goorney S. DigiQ: Digitally Enhanced Quantum Technology Master. 2023. https://www.digiq.eu/.
13. QURECA: QTIndu: Quantum Technologies Courses for Industry. 2023. https://qtindu.eu/.
14. Stadermann HKE, van den Berg E, Goedhart MJ. Analysis of secondary school quantum physics curricula of 15 different countries: different perspectives on a challenging topic. Phys Rev Phys Educ Res. 2019;15(1):010130. https://doi.org/10.1103/PhysRevPhysEducRes.15.010130.
15. Pade J. Quantum mechanics for pedestrians 1: fundamentals. Undergraduate lecture notes in physics. Cham: Springer; 2018. https://doi.org/10.1007/978-3-030-00464-4.
16. Müller R, Greinert F. Quantum technologies: for engineers. Berlin: De Gruyter Oldenbourg; 2023.
17. Bennett CH, Brassard G, Crépeau C, Jozsa R, Peres A, Wootters WK. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys Rev Lett. 1993;70(13):1895–9. https://doi.org/10.1103/PhysRevLett.70.1895.
18. Bouwmeester D, Pan J-W, Mattle K, Eibl M, Weinfurter H, Zeilinger A. Experimental quantum teleportation. Nature. 1997;390(6660):575–9. https://doi.org/10.1038/37539.
19. Niedderer H, Schecker H. Towards an explicit description of cognitive systems for research in physics learning. In: Duit R, Goldberg F, Niedderer H, editors. Research in physics learning—theoretical issues and empirical studies. 1992. p. 74–98.
20. Posner GJ, Strike KA, Hewson PW, Gertzog WA. Accommodation of a scientific conception: toward a theory of conceptual change. Sci Educ. 1982;66(2):211–27. https://doi.org/10.1002/sce.3730660207.

21. Schecker H, Wilhelm T, Hopf M, Duit R, Fischler H, Haagen-Schützenhöfer C, Höttecke D, Müller R, Wodzinski R, editors. Schülervorstellungen und Physikunterricht: Ein Lehrbuch Für Studium, Referendariat und Unterrichtspraxis. Lehrbuch. Berlin: Springer; 2018.
22. Bitzenbauer P. Quantum physics education research over the last two decades: a bibliometric analysis. Educ Sci. 2021;11(11):699. https://doi.org/10.3390/educsci11110699.
23. Bouchée T, Putter-Smits L, Thurlings M, Pepin B. Towards a better understanding of conceptual difficulties in introductory quantum physics courses. Stud Sci Educ. 2022;58(2):183–202. https://doi.org/10.1080/03057267.2021.1963579.
24. Krijtenburg-Lewerissa K, Pol HJ, Brinkman A, van Joolingen WR. Insights into teaching quantum mechanics in secondary and lower undergraduate education. Phys Rev Phys Educ Res. 2017;13(1). https://doi.org/10.1103/PhysRevPhysEducRes.13.010109.
25. Singh C, Marshman E. Review of student difficulties in upper-level quantum mechanics. Phys Rev Spec Top—Phys Educ Res. 2015;11(2). https://doi.org/10.1103/PhysRevSTPER.11.020117.
26. Styer DF. Common misconceptions regarding quantum mechanics. Am J Phys. 1996;64(1):31–4. https://doi.org/10.1119/1.18288.
27. Ubben M. Typisierung des Verständnisses Mentaler Modelle Mittels Empirischer Datenerhebung Am Beispiel der Quantenphysik. Berlin: Logos Verlag; 2020. https://doi.org/10.30819/5181.
28. Bungum B, Henriksen EK, Angell C, Tellefsen CW, Bøe MV. Relequant—improving teaching and learning in quantum physics through educational design research. Nordic Stud Sci Educ. 2015;11(2):153–68. https://doi.org/10.5617/nordina.2043.
29. Moraga-Calderón TS, Buisman H, Cramer J. The relevance of learning quantum physics from the perspective of the secondary school student: a case study. Eur J Scid Math Educ. 2020;8(1):32–50.
30. Krijtenburg-Lewerissa K, Pol HJ, Brinkman A, van Joolingen WR. Key topics for quantum mechanics at secondary schools: a delphi study into expert opinions. Int J Sci Educ. 2019;41(3):349–66. https://doi.org/10.1080/09500693.2018.1550273.
31. Weissman EY, Merzel A, Katz N, Galili I. Phenomena and principles: presenting quantum physics in a high school curriculum. Physics. 2022;4(4):1299–317. https://doi.org/10.3390/physics4040083.
32. Michelini M, Ragazzon R, Santi L, Stefanel A. Proposal for quantum physics in secondary school. Phys Educ. 2000;35(6):406–10. https://doi.org/10.1088/0031-9120/35/6/305.
33. Baily C, Finkelstein ND. Teaching quantum interpretations: revisiting the goals and practices of introductory quantum physics courses. Phys Rev Spec Top—Phys Educ Res. 2015;11(2). https://doi.org/10.1103/PhysRevSTPER.11.020124.
34. Bitzenbauer P, Meyn J-P. A new teaching concept on quantum physics in secondary schools. Phys Educ. 2020;55(5):055031. https://doi.org/10.1088/1361-6552/aba208.
35. Bondani M, Chiofalo ML, Ercolessi E, Macchiavello C, Malgieri M, Michelini M, Mishina O, Onorato P, Pallotta F, Satanassi S, Stefanel A, Sutrini C, Testa I, Zuccarini G. Introducing quantum technologies at secondary school level: challenges and potential impact of an online extracurricular course. Physics. 2022;4(4):1150–67. https://doi.org/10.3390/physics4040075.
36. Chiofalo ML, Foti C, Michelini M, Santi L, Stefanel A. Games for teaching/learning quantum mechanics: a pilot study with high-school students. Educ Sci. 2022;12(7):446. https://doi.org/10.3390/educsci12070446.
37. Fischler H, Lichtfeldt M. Modern physics and students' conceptions. Int J Sci Educ. 1992;14(2):181–90. https://doi.org/10.1080/0950069920140206.
38. Müller R, Wiesner H. Teaching quantum mechanics on an introductory level. Am J Phys. 2002;70(3):200–9. https://doi.org/10.1119/1.1435346.
39. Nagelschmidt J, Heimersheim S, Hartmann F. Quantenverschränkung low-cost: Jugend Forscht 2014. https://jufo.stmg.de/2014/Quantenverschraenkung/Quantenverschraenkung.pdf Accessed 20.06.2023.
40. Dehlinger D, Mitchell MW. Entangled photon apparatus for the undergraduate laboratory. Am J Phys. 2002;70(9):898–902. https://doi.org/10.1119/1.1498859.
41. Dehlinger D, Mitchell MW. Entangled photons, nonlocality, and bell inequalities in the undergraduate laboratory. Am J Phys. 2002;70(9):903–10. https://doi.org/10.1119/1.1498860.
42. Aspden RS, Padgett MJ, Spalding GC. Video recording true single-photon double-slit interference. Am J Phys. 2016;84(9):671–7. https://doi.org/10.1119/1.4955173.
43. Kohnle A, Baily C, Campbell A, Korolkova N, Paetkau MJ. Enhancing student learning of two-level quantum systems with interactive simulations. Am J Phys. 2015;83(6):560–6. https://doi.org/10.1119/1.4913786.
44. Bitzenbauer P. Effect of an introductory quantum physics course using experiments with heralded photons on preuniversity students' conceptions about quantum physics. Phys Rev Phys Educ Res. 2021;17(2). https://doi.org/10.1103/PhysRevPhysEducRes.17.020103.
45. Bronner P, Strunz A, Silberhorn C, Meyn J-P. Interactive screen experiments with single photons. Eur J Phys. 2009;30(2):345.
46. Marckwordt J, Muller A, Harlow D, Franklin D, Landsberg RH. Entanglement ball: using dodgeball to introduce quantum entanglement. Phys Teach. 2021;59(8):613–6. https://doi.org/10.1119/5.0019871.
47. López-Incera A, Dür W. Entangle me! A game to demonstrate the principles of quantum mechanics. Am J Phys. 2019;87(2):95–101. https://doi.org/10.1119/1.5086275.
48. Goff A. Quantum tic-tac-toe: a teaching metaphor for superposition in quantum mechanics. Am J Phys. 2006;74(11):962–73. https://doi.org/10.1119/1.2213635.
49. Kopf L, Hiekkamäki M, Prabhakar S, Fickler R. Endless fun in high dimensions—a quantum card game. Am J Phys. 2023;91(6):458. https://doi.org/10.1119/5.0062128.
50. van Nieuwenburg E. Quantum TiqTaqToe. https://quantumtictactoe.com/ Accessed 28.06.2023.
51. Kohnle A, Deffebach E. Investigating student understanding of quantum entanglement. 2015 Physics Education Research Conference Proceedings. p. 171–174. https://doi.org/10.1119/perc.2015.pr.038.
52. Mayring P, Fenzl T. Qualitative inhaltsanalyse. In: Baur N, Blasius J, editors. Handbuch Methoden der Empirischen Sozialforschung. Wiesbaden: Springer; 2019. p. 633–48.

53. Hennig F, Lipps M, Ubben MS, Bitzenbauer P. From the Big Bang to life beyond Earth: German preservice physics teachers' conceptions of astronomy and the nature of science. Educ Sci. 2023;13(5):475. https://doi.org/10.3390/educsci13050475.
54. Landis JR, Koch GG. The measurement of observer agreement for categorical data. Biometrics. 1977;33(1):159. https://doi.org/10.2307/2529310.
55. Hemphill JF. Interpreting the magnitudes of correlation coefficients. Am Psychol. 2003;58(1):78–9. https://doi.org/10.1037/0003-066X.58.1.78.
56. Einstein A, Podolsky B, Rosen N. Can quantum-mechanical description of physical reality be considered complete? Phys Rev. 1935;47(10):777–80. https://doi.org/10.1103/PhysRev.47.777.
57. Aspect A, Grangier P, Roger G. Experimental realization of Einstein-Podolsky-Rosen-Bohm gedankenexperiment: a new violation of bell's inequalities. Phys Rev Lett. 1982;49(2):91–4. https://doi.org/10.1103/PhysRevLett.49.91.
58. Clauser JF, Horne MA, Shimony A, Holt RA. Proposed experiment to test local hidden-variable theories. Phys Rev Lett. 1969;23(15):880–4. https://doi.org/10.1103/PhysRevLett.23.880.
59. Aehle S, Scheiger P, Cartarius H. An approach to quantum physics teaching through analog experiments. Physics. 2022;4(4):1241–52. https://doi.org/10.3390/physics4040080.
60. Greinert F, Bodensiek O, Essing D, Muthusamy G. Quantenteleportation und verschränkung im science center mit erweiterter realität: Projekt holodeck:q. PhyDid B—Didaktik der Physik—Beiträge zur DPG-Frühjahrstagung. 2022;1.
61. Woitzik AJC. Quanteninformationsverarbeitung in der Gymnasialen Oberstufe. 2020.
62. Greinert F, Müller R, Bitzenbauer P, Ubben MS, Weber K-A. Future quantum workforce: competences, requirements, and forecasts. Phys Rev Phys Educ Res. 2023;19:010137. https://doi.org/10.1103/PhysRevPhysEducRes.19.010137.
63. Krijtenburg-Lewerissa K, Pol HJ, Brinkman A, Joolingen WR. Secondary school students' misunderstandings of potential wells and tunneling. Phys Rev Phys Educ Res. 2020;16:010132. https://doi.org/10.1103/PhysRevPhysEducRes.16.010132.
64. Bitzenbauer P. Effect of an introductory quantum physics course using experiments with heralded photons on preuniversity students' conceptions about quantum physics. Phys Rev Phys Educ Res. 2021;17(2). https://doi.org/10.1103/PhysRevPhysEducRes.17.020103.
65. Kohnle A, Deffebach E. Investigating student understanding of quantum entanglement. https://doi.org/10.48550/arXiv.1512.02346.
66. Michelini M, Santi L, Stefanel A, et al. Building quantum formalism in upper secondary school students. Teach Learn Phys Today: Chall. 2014. 109–114.
67. Wineland DJ. Nobel lecture: superposition, entanglement, and raising Schrödinger's cat. Rev Mod Phys. 2013;85(3):1103.
68. Montagnani S, Stefanel A, Chiofalo MLM, Santi L, Michelini M. An exp eriential program on the foundations of quantum mechanics for final-year high-school students. Phys Educ. 2023;58(3):035003.

Publisher's Note