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Features of quantum thermodynamics induced by common environments based on collision model

Rui Huang[1], Zhong-Xiao Man[1*], Ying-Jie Zhang[1] and Yun-Jie Xia[1]

*Correspondence:
zxman@qfnu.edu.cn
[1]School of Physics and Physical Engineering, Shandong Provincial Key Laboratory of Laser Polarization and Information Technology, Qufu Normal University, Qufu, China

Abstract

The common reservoir can cause some unique effects, such as dark state and steady-state coherence, which are extensively studied in the dynamics of open quantum system. In this work, by means of collision model, we explore features of quantum thermodynamics induced by common reservoirs. We first construct general formulations of thermodynamic quantities for the system consisting of *N* coupling subsystems embedded in *M* common thermal reservoirs. We confirm the existence of nonlocal work due to simultaneous interactions of subsystems with the common reservoirs resembling what is found for nonlocal heat. With a system of two coupled qubits in a common reservoir, we show that steady-state currents could emerge even when interactions of individual subsystems and the reservoir fulfill strict energy conservation. We also exhibit the effect of dark state on the steady-state currents. We then examine relations between the work cost, the system's nonequilibrium steady-state and the extractable work. In particular, we find that in the presence of dark state, the work cost is only related to the coherence generated in the dynamical evolution but not to the one contributed by the initial dark state of the system. We also show the possible transformation of coherence into useful work in terms of ergotropy. We finally examine the scale effect of reservoirs and show that the increase of the number of involved reservoirs need more work to be costed and meanwhile can produce more coherence so that more ergotropy can be extracted. The obtained features contribute to the understanding of thermodynamics in common reservoirs and would be useful in quantum technologies when common reservoirs are necessary.

Keywords: Quantum thermodynamics; Common environment; Collision model

1 Introduction

Recent years have seen increasingly comprehensive combinations of researches between the theory of open quantum system (OQS) [1] and quantum thermodynamics (QT) [2, 3]. Explorations of OQS are promoted by rapid progress of quantum information technology with the purpose to achieve on-demand manipulations on the OQS subject to various destruction of environments [1]. The QT, on the other hand, examines classical thermodynamics laws in the quantum level and implement various thermodynamic tasks by using quantum systems [2–7]. Since the QT exploits OQS as working substance, the theoretical frameworks treating system-environment interactions in the theory of OQS, such as the quantum master equation (QME), are also applicable in QT. Moreover, the pursuit of quantum advantages of various quantum effects, such as quantum coherence [8–16], which have been widely studied in the theory of OQS, become a central issue in QT as well.

The QME is the most popular tool in dealing with the dynamics of OQS surrounded by external environments. Generally, QME includes the local and global forms with their jump operators acting on local subsystems and the global degrees of the system, respectively [1]. For a specific model, how to choose these two approaches relies, to some extent, on the competition between strength of inner interactions among subsystems and that of system-environment couplings. In the study of QT, although the QME is also applicable, much more attentions should be paid to these two choices to avoid the occurrence of thermodynamic inconsistences [17–30]. It is found that the local QME exhibits weakness in dealing with QT, such as being unable to reach thermal equilibrium even for weak interaction of system-environment [18, 19], missing important effects in thermal quantum devices [20, 21], and even getting results that violate the second law of thermodynamics [30]. On the other hand, the scope of application of global QME is also limited [23–26], which is found to fail to give completely positive maps (in the Redfield type) [25] and be insufficient for the depiction of heat current in stationary nonequilibrium regime [26].

Beyond the conventional QME, the collision model(CM) [31], as another popular framework in simulating dynamics of OQS [32–49], proves to be a powerful tool in the exploration of QT [50–61]. In this framework, the environment is modeled as a collection of identically prepared ancillas. At each step, the system of interest interacts, or collides, with a ancilla which is then discarded and replaced with a fresh one in the next step [31]. It has been shown that the CM is consistent with Lindblad QME when no correlations exist initially and are created in the later dynamical process [33, 34]. Due to its versatile feature, the CM is widely used in recovering the non-Markovian dynamics by, for instance, introducing either initial correlations among ancillas or ancilla-ancilla collisions in between two system-ancilla collisions [38–49]. By taking advantage of CM, one has successfully constructed the link between non-Markovianity and QT so that the exploration of thermodynamics in the non-Markovian process is made possible [62–68]. In addition to that, the CM also sheds some light on the settlement of thermodynamic inconsistences [69–72]. In Ref. [30], Levy and Kosloff have considered a network composed of two subsystems coupled to two thermal reservoirs with different temperatures. They have found that if the local QME is used for the system's dynamics, the second law of thermodynamics will be violated with heat flowing automatically from cold to hot reservoirs even in the limit of vanishing coupling between the subsystems [30]. Barra has recognized that in the CM external work is required to switch on and off the successive system-environment collisions [69]. Once the extra work cost of maintaining the successive collisions is taken into account, the local QME based on CM is shown to comply again with thermodynamic laws [71].

The requirement of work in CM become apparent when one recognizes that, in each stroke of collision, the system-environment couplings must be turned on and off which results in time-dependence of total Hamiltonian of system and environment. The quantum work is just defined as energy change due to the change of system's Hamiltonian.

Therefore, one may image that different types of system-environment couplings require different forms of work to maintain. In most situations, the environment ancilla collides locally with a system so that the definition of work in this case is straightforward [71]. If the ancilla collides simultaneously with more than one system, e.g., the system comprises several subsystems embedded in a common environment, the form of work is not clear so far. It is known that due to the indistinguishability of subsystems from the view of common environment, there will appear collective dissipative terms in the QME depicting the system's dynamics and nonlocal currents of heat between the system and environment [68, 73]. In this context, one may ask whether nonlocal current of work exists and what is the explicit form if any? Furthermore, what are thermodynamic features of quantum system induced by common environments?

Motivated by these questions, in this work, we study QT in common reservoirs through considering a model consisting of N coupling subsystems simultaneously interacting with M common reservoirs. We first construct general forms of thermodynamic quantities from their most fundamental definitions and identify their local and nonlocal components. We therefore confirm that, as a unique feature of common reservoir, the nonlocal work really exists resembling what has been found for nonlocal heat [68, 73]. The results are demonstrated via a system that consists of two qubits embedded in a common reservoir. Aiming at the peculiarity of common reservoir, we discuss the condition for the appearance of steady-state currents and influences of dark state on the currents. We then concentrate on relations between the amount of work cost, the system's nonequilibrium steady-state (NESS), and the extractable work in terms of ergotropy [74]. In particular, we find that the work cost cannot be directly connected to the total coherence that characterizes the NESS of the system if the dark state arises. We also exhibit the role of steady-state coherence in facilitating the extraction of useful work from the system. Finally, we study the scaling effect of multiple reservoirs on the steady-state currents, the coherence and the ergotropy.

This paper is structured as follows. We start by introducing the CM in common reservoir and derive the QME governing system's dynamics in Sect. 2. In Sect. 3, we formulate thermodynamics quantities, i.e., the heat, work and internal energy, from their most fundamental definition and identify the local and nonlocal components of heat and work. In Sect. 4, focusing on the system of two coupled qubits, we demonstrate the steady-state currents of work and heat and the effect of dark state on them. In Sect. 5, we show relations of work cost, NESS of the system, and the extractable work and discuss the scale effect of multiple reservoirs. Finally, conclusions are given in Sect. 6.

2 The model and master equation

The system S we consider consists of N subsystems $S_1, S_2, \ldots, S_N$ surrounded by M reservoirs $R_1, R_2, \ldots, R_M$ and each one of the reservoirs is common to the whole system. The scenario of common environment can arise naturally if subsystems are sufficiently close to each other being indistinguishable from the aspect of environment. Moreover, since many novel quantum effects can be induced by common environments, such as superradiance and decoherence-free subspace, one also engineer common environments in the laboratory [75, 76]. In experiment, the dynamics and/or thermodynamics of a system in common environment can be realized in different platforms, such as atoms, ions or quantum dots in a (leaky) cavity and superconducting qubits in correlated electronic noise and so on [77–82]. In this context, it is useful to provide a consistent description of thermodynamics in the presence of common environments by means of CM. Within the framework of CM, the reservoir R_k (with $k = 1, 2, \dots, M$) is modeled as a series of identically prepared ancillas and at each time an ancilla interacts/collides simultaneously with the N subsystems for a short duration τ. The ancilla after collision is then replaced by a new one and the process is iterated. For convenience, we use the notation R_k to refer to both the kth reservoir and the generic ancilla therein. In the presence of intrasystem couplings, the Hamiltonian of the system is given as

$$\hat{H}_S = \sum_{i=1}^{N} \hat{H}_{S_i} + \hat{H}_I, \tag{1}$$

where $\hat{H}_{S_i}$ is the free Hamiltonian of the subsystem S_i and $\hat{H}_I$ summarizes all interactions between subsystems. The total Hamiltonian of the system plus the reservoirs takes the form

$$\hat{H}_{\mathrm{tot}} = \hat{H}_S + \sum_{k=1}^{M} \hat{H}_{R_k} + \frac{1}{\sqrt{\tau}} \sum_{k=1}^{M} \sum_{i=1}^{N} \hat{V}_{ik}, \tag{2}$$

where $\hat{H}_{R_k}$ is the free Hamiltonian of reservoir R_k and $\hat{V}_{ik}$ stands for the interaction between S_i and R_k. To be convenient for taking continuous time limit later, we have scaled $\hat{V}_{ik}$ with $1/\sqrt{\tau}$ although not necessary.

We first establish QME to describe the dynamics of the system. After a collision of duration τ, the state ρ_S of the system at time t will be transformed to ρ'_S at time $t + \tau$ as

$$\rho'_S = \mathrm{tr}_R\{\hat{U}_{SR}\rho_{SR}\hat{U}^{\dagger}_{SR}\}, \tag{3}$$

where $\hat{U}_{SR} = e^{-i\tau\hat{H}_{\mathrm{tot}}}$ is the unitary time evolution operator and $\rho_{SR} = \rho_S \otimes \rho_R$ with ρ_R the total state of the reservoirs. We assume that the reservoir R_k is prepared in the thermal state $\rho^{th}_{R_k} = e^{-\beta_k\hat{H}_{R_k}}/Z_{R_k}$ so that $\rho_R = \prod_{k=1}^{M} \rho^{th}_{R_k}$ with $\beta_k = 1/T_k$ the inverse temperature and $Z_{R_k} = \mathrm{tr}\{e^{-\beta_k\hat{H}_{R_k}}\}$ the corresponding partition function. We have set $\hbar = k_B = 1$ here and throughout the paper. By expanding $\hat{U}_{SR}$ up to the first order in τ, we get QME governing the dynamics of the system as

$$\begin{aligned} \frac{d\rho_S}{dt} &= \lim_{\tau\to 0}\big[(\rho'_S - \rho_S)/\tau\big] \\ &= -i[\hat{H}_S, \rho_S] + \sum_{k=1}^{M} \mathcal{D}_k(\rho_S), \end{aligned} \tag{4}$$

where

$$\mathcal{D}_k(\rho_S) = -\frac{1}{2}\sum_{i,j=1}^{N} \mathrm{Tr}_R\big[\hat{V}_{ik}, [\hat{V}_{jk}, \rho_S\rho_R]\big]. \tag{5}$$

Note that in Eq. (5), the values i and j can be the same or different with the later case, i.e., $i \neq j$, represents the collective dissipations due to the indistinguishability of the subsystems from view of the common reservoir.

3 Thermodynamics quantities with local and nonlocal components

In the following, we formulate thermodynamics quantities in common reservoirs from their most fundamental definitions thanks to the CM approach. In particular, we identify the nonlocal component of quantum work resembling what has been found for the nonlocal heat in common environment [73]. For convenience, we denote $\rho'_{SR} = \hat{U}_{SR}\rho_{SR}\hat{U}^{\dagger}_{SR}$ as state of the system and reservoirs after a single collision. The expectation value of any observable $\hat{O}$ of either the system or the reservoir with respect to ρ'_{SR} after the collision can be expressed as $\langle \hat{O}\rangle_{\rho'_{SR}} = \langle \hat{U}^{\dagger}_{SR}\hat{O}\hat{U}_{SR}\rangle_{\rho_{SR}}$ with $\langle\cdot\rangle_{\rho} = \mathrm{tr}[\cdot\rho]$, which we shall use repeatedly.

In a single collision, the heat flowed to the system from the reservoir R_k can be defined as its energy decrease as

$$\begin{aligned}\Delta Q_k &= \langle \hat{H}_{R_k}\rangle_{\rho_{SR}} - \langle \hat{H}_{R_k}\rangle_{\rho'_{SR}} \\ &= \Delta Q_k^{\mathrm{loc}} + \Delta Q_k^{\mathrm{non-loc}},\end{aligned} \tag{6}$$

where

$$\Delta Q_k^{\mathrm{loc}} = \frac{\tau}{2}\sum_{i=1}^{N}\big\langle\big[\hat{V}_{ik},[\hat{V}_{ik},\hat{H}_{R_k}]\big]\big\rangle_{\rho_{SR}} \tag{7}$$

and

$$\Delta Q_k^{\mathrm{non-loc}} = \frac{\tau}{2}\sum_{\substack{i,j=1\\(i\neq j)}}^{N}\big\langle\big[\hat{V}_{jk},[\hat{V}_{ik},\hat{H}_{R_k}]\big]\big\rangle_{\rho_{SR}}. \tag{8}$$

From Eq. (6) one can see that the heat is contributed by two components with $\Delta Q_k^{\mathrm{loc}}$ the local heat like the situation that an individual subsystem is coupled to R_k, while $\Delta Q_k^{\mathrm{non-loc}}$ the nonlocal heat induced by the simultaneous interactions of the subsystems with R_k.

The quantum work is generally defined as a change of internal energy due to the change of Hamiltonian of the system. In the CM, the system should be successively coupled to and decoupled from reservoirs leading to the time-dependence of the total Hamiltonian. Therefore, the energetic cost to retain these sequential collisions is contributed by the work. The definition of work is straightforward for the situation where the reservoir collides locally with a single system [71]. For the common reservoir considered here, however, the reservoir collides simultaneously with all the subsystems. The question of how the work maintains such type of collisions, or in other words, what is the form of the work in this case, is one of our main concerns. We show that being similar to nonlocal heat, the work in the common reservoir also exhibits the non-locality.

Since the system and reservoirs as a whole undergo unitary dynamics, the work of a single collision that occurs within the time interval $[t, t+\tau]$ is defined as

$$\Delta W = \int_t^{t+\tau}\mathrm{Tr}_{SR}\left[\frac{\partial \hat{H}_{\mathrm{tot}}}{\partial s}\rho_{SR}\right]ds. \tag{9}$$

In the Hamiltonian $\hat{H}_{\text{tot}}$ (2), only the term $\hat{V}_{ik}$ is time-dependent, so that an integration over (9) yields the concrete form of work as

$$\Delta W = \Delta W^{\text{loc}} + \Delta W^{\text{non-loc}}, \tag{10}$$

with

$$\Delta W^{\text{loc}} = -\frac{\tau}{2}\sum_{i=1}^{N}\sum_{k=1}^{M}\big\langle\big[\hat{V}_{ik},[\hat{V}_{ik},\hat{H}_{S_i}+\hat{H}_{R_k}+\hat{H}_I]\big]\big\rangle_{\rho_{SR}}, \tag{11}$$

and

$$\Delta W^{\text{non-loc}} = -\frac{\tau}{2}\sum_{\substack{i,j=1\\(i\neq j)}}^{N}\sum_{k=1}^{M}\big\langle\big[\hat{V}_{jk},[\hat{V}_{ik},\hat{H}_{S_i}+\hat{H}_{R_k}+\hat{H}_I]\big]\big\rangle_{\rho_{SR}}. \tag{12}$$

By Eqs. (11) and (12), we have divided the total work into local and nonlocal components. The formulation (11) indicates that the local work coincides with the one that would be obtained if each individual subsystem were in contact locally with the reservoirs in the absence of other ones. The nonlocal work (12), as a unique feature for the common reservoir, is identified by the appearance of crossing terms of interaction Hamiltonians in terms of $\hat{V}_{ik}$ and $\hat{V}_{jk}$ with $i \neq j$. The local and nonlocal work thus play the roles of maintaining the local and collectively collisions between subsystems with the common reservoirs, respectively.

As mentioned previously, the work is energetic cost to successively switch on/off the collisions of system and reservoir. If collisions happen between a single quantum system and reservoir, no work is needed once the collisions can preserve the energy of system and reservoir. However, it is not always true when the system consists of several subsystems with inner interactions. Our formulations (11) and (12) clearly indicate that even when the collisions in terms of $\hat{V}_{ik}$ preserves the local energy of the individual subsystem S_i and reservoir R_k, i.e., $[\hat{V}_{ik},\hat{H}_{S_i}+\hat{H}_{R_k}] = 0$, for all $1 \leq i \leq N$ and $1 \leq k \leq M$, a finite nonzero work might still exist due to the existence of intrasystem interaction $\hat{H}_I$. In this case the local and nonlocal work are reduced respectively to

$$\Delta W_I^{\text{loc}} = -\frac{\tau}{2}\sum_{i=1}^{N}\sum_{k=1}^{M}\big\langle\big[\hat{V}_{ik},[\hat{V}_{ik},\hat{H}_I]\big]\big\rangle_{\rho_{SR}}, \tag{13}$$

and

$$\Delta W_I^{\text{non-loc}} = -\frac{\tau}{2}\sum_{\substack{i,j=1\\(i\neq j)}}^{N}\sum_{k=1}^{M}\big\langle\big[\hat{V}_{jk},[\hat{V}_{ik},\hat{H}_I]\big]\big\rangle_{\rho_{SR}}. \tag{14}$$

In the Sect. 4, with a concrete model, we shall demonstrate the currents of work and heat as well as their local and nonlocal contributions. In particular, we address the types of intrasystem interactions that can result in nonzero steady-state current of work even the interactions of individual subsystems and reservoir satisfy energy-conservation.

Finally, we derive the change of internal energy of the system as

$$\begin{aligned}\Delta E &= \langle \hat{H}_S\rangle_{\rho'_{SR}} - \langle \hat{H}_S\rangle_{\rho_{SR}} \\ &= -\frac{\tau}{2}\sum_{i=1}^{N}\sum_{k=1}^{M}\big\langle\big[\hat{V}_{ik},[\hat{V}_{ik},\hat{H}_{S_i}+\hat{H}_I]\big]\big\rangle_{\rho_{SR}} \\ &\quad -\frac{\tau}{2}\sum_{\substack{i,j=1\\(i\neq j)}}^{N}\sum_{k=1}^{M}\big\langle\big[\hat{V}_{jk},[\hat{V}_{ik},\hat{H}_{S_i}+\hat{H}_I]\big]\big\rangle_{\rho_{SR}}.\end{aligned} \tag{15}$$

From Eqs. (6), (10) and (15), we can obtain $\Delta E = \Delta W + \sum_{k=1}^{M}\Delta Q_k$ implying that our derived quantities fulfill the first law of thermodynamics.

4 Demonstration by two coupled qubits

To demonstrate our results, we consider that the system consists of a pair of two-level subsystems (qubits) S_1 and S_2 embedded in M common reservoirs. The free Hamiltonian of S_i ($i = 1,2$) is given as $\hat{H}_{S_i} = \frac{\omega_{S_i}}{2}\hat{\sigma}^z_{S_i}$ with frequency ω_{S_i} and $\{\hat{\sigma}^x_A,\hat{\sigma}^y_A,\hat{\sigma}^z_A\}$ the usual Pauli operators for the qubit A. The generic ancilla in the reservoir R_k ($k = 1,2,\dots,M$) is also modeled as qubit with the Hamiltonian $\hat{H}_{R_k} = \frac{\omega_{R_k}}{2}\hat{\sigma}^z_{R_k}$ with frequency ω_{R_k}. We assume that the ancillas of reservoir R_k are prepared in thermal states

$$\rho_{R_k} = \big[(1+\xi_{R_k})/2\big]|0\rangle\langle 0| + \big[(1-\xi_{R_k})/2\big]|1\rangle\langle 1|, \tag{16}$$

where $\xi_{R_k} = \tanh(\beta_k\omega_{R_k}/2)$ with $\beta_k = 1/T_k$ the inverse temperature and $|0\rangle(|1\rangle)$ denotes the ground (excited) state of R_k. We consider the XY-type interactions for subsystems S_1 and S_2

$$\hat{H}_{S_1S_2} = J^x_s\hat{\sigma}^x_{S_1}\hat{\sigma}^x_{S_2} + J^y_s\hat{\sigma}^y_{S_1}\hat{\sigma}^y_{S_2}, \tag{17}$$

and for collisions between S_i and R_k

$$\hat{V}_{ik} = J^x_{ik}\hat{\sigma}^x_{S_i}\hat{\sigma}^x_{R_k} + J^y_{ik}\hat{\sigma}^y_{S_i}\hat{\sigma}^y_{R_k}, \tag{18}$$

in which $J^{x(y)}_s$ and $J^{x(y)}_{ik}$ denote the interaction strengths.

The system's dynamics is described by the master equation (4) with

$$\hat{H}_S = \hat{H}_{S_1} + \hat{H}_{S_2} + \hat{H}_{S_1S_2} \tag{19}$$

and

$$\begin{aligned}\mathcal{D}_k(\rho_S) = \sum_{i,j=1}^{2} J^x_{ik}J^x_{jk}&\left(\hat{\sigma}^x_{S_i}\rho_S\hat{\sigma}^x_{S_j} - \frac{1}{2}\big[\hat{\sigma}^x_{S_j}\hat{\sigma}^x_{S_i},\rho_S\big]_+\right) \\ &+ J^y_{ik}J^y_{jk}\left(\hat{\sigma}^y_{S_i}\rho_S\hat{\sigma}^y_{S_j} - \frac{1}{2}\big[\hat{\sigma}^y_{S_j}\hat{\sigma}^y_{S_i},\rho_S\big]_+\right) \\ &+ i\big\langle\hat{\sigma}^z_{R_k}\big\rangle_{\rho_{R_k}}\left[J^y_{ik}J^x_{jk}\left(\hat{\sigma}^y_{S_i}\rho_S\hat{\sigma}^x_{S_j} - \frac{1}{2}\big[\hat{\sigma}^x_{S_j}\hat{\sigma}^y_{S_i},\rho_S\big]_+\right)\right.\end{aligned}$$

$$-J_{ik}^{x}J_{jk}^{y}\left(\hat{\sigma}_{S_i}^{x}\rho_S\hat{\sigma}_{S_j}^{y}-\frac{1}{2}\left[\hat{\sigma}_{S_j}^{y}\hat{\sigma}_{S_i}^{x},\rho_S\right]_+\right)\Bigg], \tag{20}$$

with $[\ldots,\ldots]_+$ the anticommutator.

By means of Eqs. (6)–(8) and (10)–(12) and after taking continuous time limit, we derive the current of heat with respect to the reservoir R_k, i.e., $\dot{Q}_k=\lim_{\tau\to 0}(\Delta Q_k/\tau)$, as well as its local and nonlocal components

$$\begin{aligned}\dot{Q}_k^{\mathrm{loc}}&=\lim_{\tau\to 0}\left(\Delta Q_k^{\mathrm{loc}}/\tau\right)\\&=\sum_{i=1}^{2}\omega_{R_k}\left[-2J_{ik}^{x}J_{ik}^{y}\langle\hat{\sigma}_{S_i}^{z}\rangle_{\rho_S}+\left(\left(J_{ik}^{x}\right)^2+\left(J_{ik}^{y}\right)^2\right)\langle\hat{\sigma}_{R_k}^{z}\rangle_{\rho_{R_k}}\right],\end{aligned} \tag{21}$$

and

$$\begin{aligned}\dot{Q}_k^{\mathrm{non-loc}}&=\lim_{\tau\to 0}\left(\Delta Q_k^{\mathrm{non-loc}}/\tau\right)\\&=\omega_{R_k}\langle\hat{\sigma}_{R_k}^{z}\rangle_{\rho_{R_k}}\sum_{\substack{i,j=1\\(i\neq j)}}^{2}\left[J_{ik}^{x}J_{jk}^{x}\langle\hat{\sigma}_{S_i}^{x}\hat{\sigma}_{S_j}^{x}\rangle_{\rho_S}+J_{ik}^{y}J_{jk}^{y}\langle\hat{\sigma}_{S_i}^{y}\hat{\sigma}_{S_j}^{y}\rangle_{\rho_S}\right],\end{aligned} \tag{22}$$

and that of work, i.e., $\dot{W}=\lim_{\tau\to 0}(\Delta W/\tau)$, as well as its local and nonlocal components

$$\begin{aligned}\dot{W}^{\mathrm{loc}}&=\lim_{\tau\to 0}\left(\Delta W^{\mathrm{loc}}/\tau\right)\\&=\sum_{k=1}^{M}\sum_{i=1}^{2}\left[-\left(\left(J_{ik}^{x}\right)^2+\left(J_{ik}^{y}\right)^2\right)\left(\omega_{R_k}\langle\hat{\sigma}_{R_k}^{z}\rangle_{\rho_{R_k}}+\omega_{S_i}\langle\hat{\sigma}_{S_i}^{z}\rangle_{\rho_S}\right)\right.\\&\quad\left.+2J_{ik}^{x}J_{ik}^{y}\left(\omega_{R_k}\langle\hat{\sigma}_{S_i}^{z}\rangle_{\rho_S}+\omega_{S_i}\langle\hat{\sigma}_{R_k}^{z}\rangle_{\rho_{R_k}}\right)\right]\\&\quad-\sum_{\substack{i,j=1\\(i\neq j)}}^{2}\left[J_s^{x}\left(\left(J_{ik}^{y}\right)^2+\left(J_{jk}^{y}\right)^2\right)\langle\hat{\sigma}_{S_i}^{x}\hat{\sigma}_{S_j}^{x}\rangle_{\rho_S}\right.\\&\quad\left.+J_s^{y}\left(\left(J_{ik}^{x}\right)^2+\left(J_{jk}^{x}\right)^2\right)\langle\hat{\sigma}_{S_i}^{y}\hat{\sigma}_{S_j}^{y}\rangle_{\rho_S}\right],\end{aligned} \tag{23}$$

and

$$\begin{aligned}\dot{W}^{\mathrm{non-loc}}&=\lim_{\tau\to 0}\left(\Delta W^{\mathrm{non-loc}}/\tau\right)\\&=\sum_{k=1}^{M}\sum_{\substack{i,j=1\\(i\neq j)}}^{2}\left[\langle\hat{\sigma}_{R_k}^{z}\rangle_{\rho_{R_k}}\langle\hat{\sigma}_{S_i}^{x}\hat{\sigma}_{S_j}^{x}\rangle_{\rho_S}\left(\omega_{S_i}J_{ik}^{y}J_{jk}^{x}-\omega_{R_k}J_{ik}^{x}J_{jk}^{x}\right)\right.\\&\quad+\langle\hat{\sigma}_{R_k}^{z}\rangle_{\rho_{R_k}}\langle\hat{\sigma}_{S_i}^{y}\hat{\sigma}_{S_j}^{y}\rangle_{\rho_S}\left(\omega_{S_i}J_{ik}^{x}J_{jk}^{y}-\omega_{R_k}J_{ik}^{y}J_{jk}^{y}\right)\\&\quad+2\langle\hat{\sigma}_{S_i}^{z}\hat{\sigma}_{S_j}^{z}\rangle_{\rho_S}\left(J_s^{x}J_{ik}^{y}J_{jk}^{y}+J_s^{y}J_{ik}^{x}J_{jk}^{x}\right)\\&\quad\left.-2\langle\hat{\sigma}_{R_k}^{z}\rangle_{\rho_{R_k}}\langle\hat{\sigma}_{S_i}^{z}\rangle_{\rho_S}\left(J_s^{x}J_{ik}^{y}J_{jk}^{x}+J_s^{y}J_{ik}^{x}J_{jk}^{y}\right)\right].\end{aligned} \tag{24}$$

4.1 The work due to inner interactions of subsystems

It has been recognized that, in the CM, a certain amount of work should be invested to maintain successive collisions between the system and reservoir. If the collisions guarantee strict energy conservation for the system and reservoir, then no work is needed in the steady-state and the currents of heat and work will vanish [72]. By contrast, for the system consisting of several coupling subsystems with each one colliding with its own independent reservoir, due to inner interactions between the subsystems, work cost might still be required even when collisions preserve the energy of a subsystem and its local reservoir [71]. These results make one further ask what happens for the configuration of several coupling subsystems embedded in a common reservoir. Here, we shall show that conditioned on the types of inner interactions the work cost would still be required in the stationary regime even when the energy conservation holds for each subsystem and the reservoir.

For this purpose, we consider the condition of energy conservation for collisions of S_i and a single reservoir R, namely, $J_i^x = J_i^y = J_i$ and $\omega_R = \omega_{S_i}$ for $i = 1, 2$, i.e., $[\hat{V}_i, \hat{H}_{S_i} + \hat{H}_R] = 0$. Here and henceforth, the subscript k in V_{ik}, R_k, $J_{ik}^{x(y)}$ and $\dot{Q}_k$ is omitted when only a single common reservoir is involved. Under the condition of energy conservation, the energy that leaves the subsystem S_i will enter the reservoir R, therefore one might think that no work is needed to maintain the collisions and the currents of heat and work will vanish at the steady state. However, due to the inner interactions of S_1 and S_2, the conservation of global energy of the system, namely, $[\hat{V}_i, \hat{H}_S + \hat{H}_R] = [\hat{V}_i, \hat{H}_{S_1S_2}] = 0$, cannot be ensured. Therefore, there could be finite nonzero currents of heat and work in stationary regime even when the local energy of S_i and R are conserved.

We find that whether stationary currents exist or not depends on forms of inner interactions of S_1 and S_2, namely, not all the interactions can lead to currents of work and heat in the stationary regime. If the interaction of $S_1 - S_2$ adopts the form $J_s^x = J_s^y$ and at the same time no dark state (which is to be discussed in the next subsection) appears with $J_1 \neq J_2$, both the two subsystems will arrive at ESS at the same temperature β_R as the reservoir

$$\widetilde{\rho}_{\mathrm{ESS}} = \frac{e^{-\beta_R \hat{H}_{S_1}}}{Z_1} \otimes \frac{e^{-\beta_R \hat{H}_{S_2}}}{Z_2}, \tag{25}$$

with $Z_i = \mathrm{Tr}[e^{-\beta_R \hat{H}_{S_i}}]$. In this case, all the currents of heat and work will vanish in the stationary regime. Otherwise, the system reaches NESS and nonzero currents of heat and work would come into being. In other words, a finite nonzero work should be invested to maintain the NESS of the system. This is verified in Fig. 1, where we plot the currents of heat and work as well as their local and nonlocal components, i.e., $\dot{Q}^{\mathrm{loc}}$, $\dot{Q}^{\mathrm{non-loc}}$, $\dot{W}^{\mathrm{loc}}$ and $\dot{W}^{\mathrm{non-loc}}$, as a function of J_s^x/J_s^y under the condition of $[\hat{V}_i, \hat{H}_{S_i} + \hat{H}_R] = 0$ for $i = 1, 2$ as well as $J_1 \neq J_2$. Clearly, apart from the point of $J_s^x/J_s^y = 1$, all the quantities keep finite nonzero values with the first law of thermodynamics $\dot{Q}^{\mathrm{loc}} + \dot{Q}^{\mathrm{non-loc}} + \dot{W}^{\mathrm{loc}} + \dot{W}^{\mathrm{non-loc}} = 0$ (i.e., $\dot{Q} + \dot{W} = 0$) being always satisfied. As a unique feature of the common reservoir, the nonlocal work appears and takes opposite direction to the local and total work.

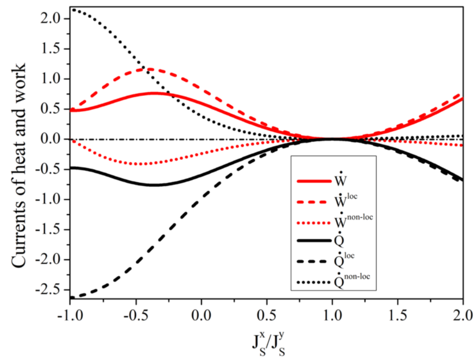

Figure 1 The currents of heat $\dot{Q}$ and work $\dot{W}$ as well as their local and nonlocal components, i.e., $\dot{W}^{\mathrm{loc}}$, $\dot{W}^{\mathrm{non-loc}}$, $\dot{Q}^{\mathrm{loc}}$ and $\dot{Q}^{\mathrm{non-loc}}$, as a function of J_s^x/J_s^y for $\omega_{S_1} = \omega_{S_2} = \omega_R = 3$, $J_1 = 0.4$, $J_2 = 1.2$ and $T_R = 1$

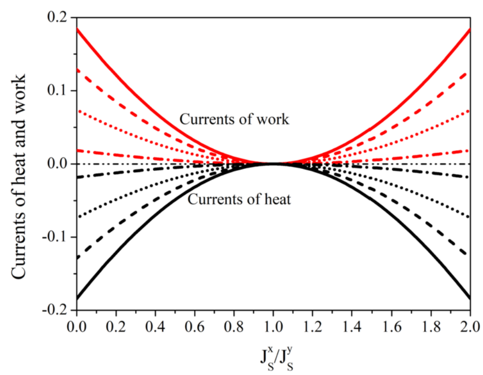

Figure 2 The currents of heat $\dot{Q}$ (black curves) and work $\dot{W}$ (red curves) as a function of J_s^x/J_s^y for different proportions P_D of the dark state, i.e., $P_D = 0$ (solid curve), $P_D = 0.3$ (dashed curve), $P_D = 0.6$ (dotted curve) and $P_D = 0.9$ (dash-dotted curve). The other parameters are set as $\omega_{S_1} = \omega_{S_2} = \omega_R = 3$, $J_1 = J_2 = 1.2$ and $T_R = 1$

4.2 The effect of dark state

A striking feature of the dynamics of two subsystems in a common reservoir is the emergence of dark state that is immune to the influence of the reservoir, namely, the proportion of dark state in the initial state of the system remains unchanged in the stationary regime. The effect of dark state on the currents of work and heat is one of our main concerns. For our model, we find that the state $|\psi\rangle_D = \frac{1}{\sqrt{2}}(|01\rangle_{S_1S_2} - |10\rangle_{S_1S_2})$ [with $|0\rangle$ ($|1\rangle$) the ground (excited) state of S_i] is the dark state of the system arising when $J_1^x = J_2^x$, $J_1^y = J_2^y$ and $\omega_{S_1} = \omega_{S_2}$. In Fig. 2, for a single common reservoir with the condition $J_1^x = J_1^y \equiv J_1 = J_2^x = J_2^y \equiv J_2$, we demonstrate the total currents of heat and work in the steady-state regime with respect to J_s^x/J_s^y for different proportion of $|\psi\rangle_D$ denoted as P_D in the system's initial state. One can observe that for any given values of $J_s^x/J_s^y \neq 1$, both the currents of work and heat shrink for an increase of P_D. This implies that the currents of work and heat are closely related to the initial preparation of the system and the more proportion of $|\psi\rangle_D$ in the system's initial state, the less work is required to maintain the collisions. For the extreme case of $P_D = 1$, namely, the system is initially prepared in the state $|\psi\rangle_D$ without any further evolution taking place, both the currents of work and heat will vanish.

5 Work cost in sustaining non-equilibrium steady-state and extractable work

So far, we have shown that when the system reaches ESS, the currents of work and heat will completely vanish, otherwise a finite nonzero work would be required to maintain the NESS. Here, we are interested in relation between the extent of system's steady-state deviating from equilibrium and the amount of work cost. We show that, as expected, the larger the system deviates from equilibrium, the more the work to be invested. Then, we further consider relation between extractable work in terms of ergotropy and coherence of the system. The consistent variations between these two parameters prove the role of coherence in facilitating the extraction of useful work from the system.

5.1 The relation between work cost and NESS

In this subsection, we address relation between the extent of deviation of the system from ESS and the amount of work cost. Upon reaching ESSs with the total state $\widetilde{\rho}_{\mathrm{ESS}}$ (25), the two subsystems S_1 and S_2 have the same temperatures β_R as the reservoir and no any correlations exist between them. Hence, the temperature bias with respect to β_R and the production of correlation can indicate the system reaching NESS. It is reasonable for us to adopt effective temperatures of the individual subsystems as well as the coherence of the system as figures of merit to characterize the extent of nonequilibrium of the system's steady-state. For a qubit S with the frequency ω_S, the effective temperature can be expressed as $T_S = \frac{\omega_S}{\ln(p_g/p_e)}$ where p_g and p_e are the populations of ground and excited states of the qubit. The coherence of the system can be quantified by l_1-norm defined as $\mathcal{C} = \sum_{l\neq m} |\rho_{lm}|$, namely, the sum of absolute values of all the off-diagonal elements of the system's state [83]. The deviation of a state from equilibrium is also quantified through the so-called entropy production rate [84] defined as $\dot{\Sigma} = \dot{S}_S - \beta_R \dot{Q}$ with $\dot{S}_S$ the change rate of the entropy of the system. In the steady-state $\dot{S}_S = 0$ one has $\dot{\Sigma} = -\beta_R \dot{Q}$ so that thermal equilibrium occurs only when $\dot{\Sigma} = \dot{Q} = 0$, whereas a NESS is characterized by a finite and constant entropy production rate. We shall show that the farther the system deviates from ESS in terms of entropy production and related quantities, the more work an external agent should supply.

As a demonstration, we consider two subsystems embedded in a single common reservoir and plot in Fig. 3(a), (b), (c) and (d), the coherence of the system, the ratio T_{S_1}/T_R, the entropy production rate, and currents of work and heat, respectively, against J_s^x/J_s^y for different T_R. We have set $\omega_{S_1} = \omega_{S_2} = \omega_R$, $J_1^x = J_1^y = J_1$ and $J_2^x = J_2^y = J_2$ but $J_1 \neq J_2$, under which the total system can reach the ESS $\widetilde{\rho}_{\mathrm{ESS}}$ (25) if and only if $J_s^x = J_s^y$ is further fulfilled. As expected, at the point of $J_s^x/J_s^y = 1$, irrespective of the values of T_R, the coherence and entropy production rate remain zero, while the temperature of S_1 (the same for S_2, but not shown here) is equal to that of R with $T_{S_1}/T_R = 1$, as demonstrated in Fig. 3(a), (c) and (b), respectively. The currents of work and heat correspondingly vanish at this point as shown in Fig. 3(d). By taking the ESS state $\widetilde{\rho}_{\mathrm{ESS}}$ as a benchmark, the work cost is expected to be proportional to the extent of deviation from it. This is actually verified by Fig. 3, where we just observe that the variations of work currents (the same for heat currents) are consistent with the coherence, the ratio T_{S_1}/T_R as well as entropy production rate that characterize deviations of the system from ESS. In particular, for a given $J_s^x/J_s^y \neq 1$, the lower the temperatures T_R, the larger the coherence, the ratio T_{S_1}/T_R and the entropy production rate, namely, the bigger the deviation of the system from ESS, which in turn lead to the larger currents of work and heat.

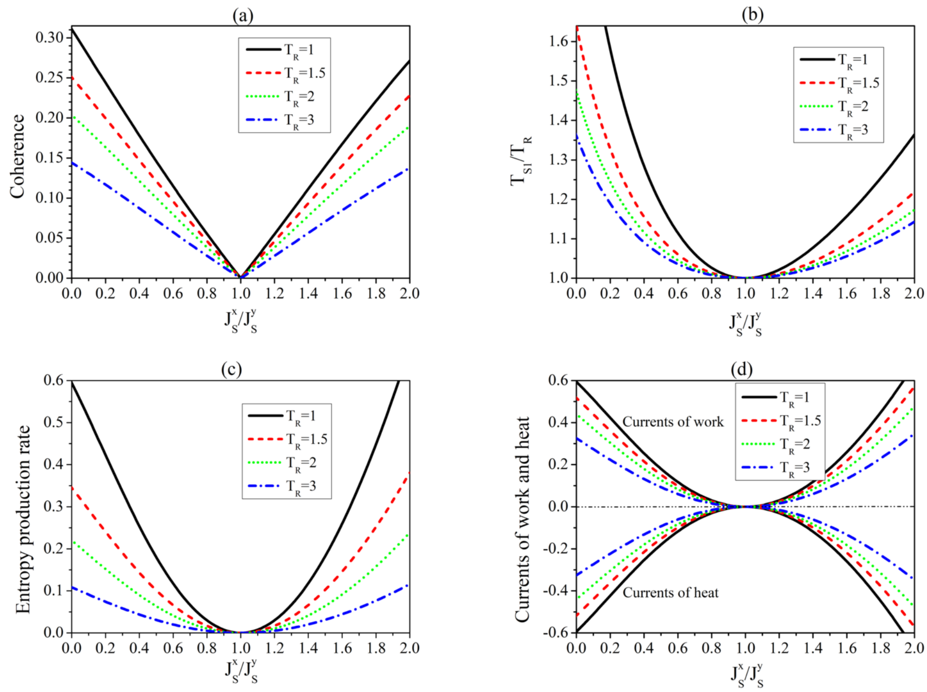

Figure 3 The coherence of S_1 and S_2 (**a**), the ratio of effective temperature of S_1 to that of the environment (**b**), entropy production (**c**), and currents of work and heat (**d**) against J^x_S/J^y_S for different temperatures T_R of the environment. The other parameters are set as $\omega_{S_1} = \omega_{S_2} = \omega_R = 3$, $J_1 = 0.4$ and $J_2 = 1.2$

Next, we consider the scenario with appearance of the dark state $|\psi\rangle_D = \frac{1}{\sqrt{2}}(|01\rangle_{S_1S_2} - |10\rangle_{S_1S_2})$ and address, in this case, the relation between work cost and the extent of nonequilibrium of the system's steady-state characterized by the coherence of the system. A striking feature of the present situation is that the steady-state of the system, so does its coherence, is related to the proportion of dark state contained in the initial state. In other words, the steady-state coherence includes two aspects, i.e., the one from dark state and the one generated in the dynamics process. Since the dark state is immune to the dynamical evolution, it is reasonable for us to suppose that the coherence of dark state does not need the support of work. To prove this, we should distinguish the coherence generated in the dynamics and that contained in the dark state. For this purpose, we subtract the component of dark state from the steady-state of the system and derive the corresponding coherence, which is labeled as generated coherence in order to distinguish with the total coherence. As a demonstration with a single common reservoir, in Fig. 4(a), (b) and (c), we plot the currents of work and heat, the total coherence, and the generated coherence, respectively, as a function of J^x_R/J^y_R for different proportions P_D of the dark state. Here, we have chosen $\omega_{S_1} = \omega_{S_2} = \omega_R$, $J^x_s = J^y_s$, $J^x_1 = J^x_2 = J^x_R$ and $J^y_1 = J^y_2 = J^y_R$. A comparison between Fig. 4(a) and (b) shows that the variations of currents are not consistent with that of the total coherence. In particular, we note that the larger the values of P_D, the larger the total steady-state coherence, but the smaller the currents. However, after subtracting the contributions of dark state, as shown in Fig. 4(c) and 4(a), the generated coherence become consistent again with the currents of woke and heat. On the one hand, with a given proportion P_D of dark state, the generated coherence and the currents of work and heat display the same increasing trends across the point $J^x_R/J^y_R = 1$. On the

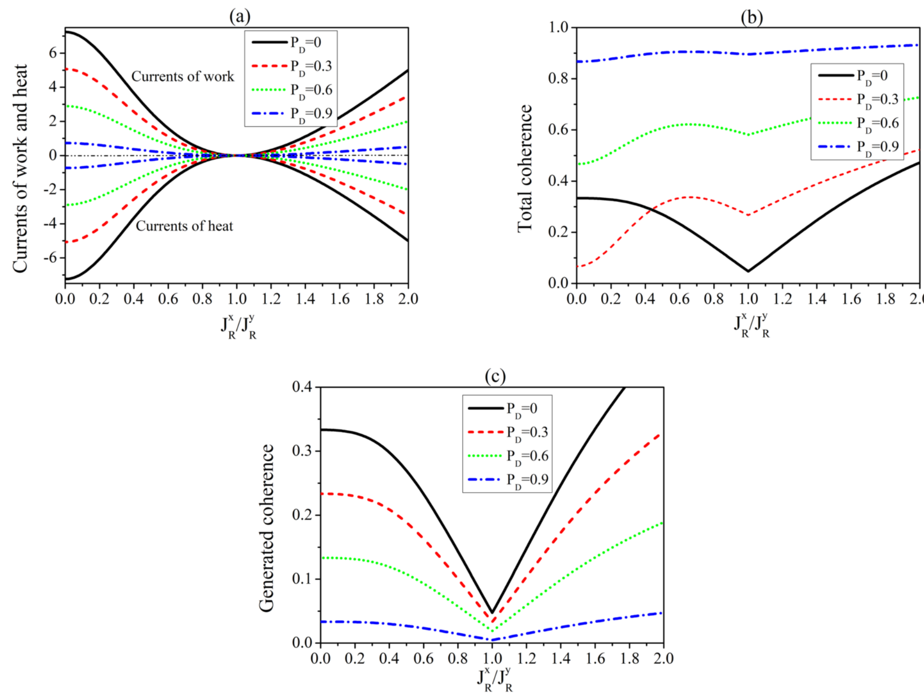

Figure 4 The currents of work and heat (**a**), total coherence (**b**), and the generated coherence (**c**) against J_R^x/J_R^y for different proportions P_D of the dark state. The other parameters are set as $\omega_{S_1} = \omega_{S_2} = \omega_R = 3$, $J_S^x = J_S^y = 1$, $J_1^x = J_2^x = J_R^x$, $J_1^y = J_2^y = J_R^y$ and $T_R = 1$

other hand, for a given $J_R^x/J_R^y \neq 1$, the less proportion of the dark state, the more generated coherence in the dynamics and the more currents of work and heat to be invested. Therefore, it is not right for one to naively connect the work cost to the total NESS of the system, which is actually only related to the component of NESS generated in the dynamical process.

5.2 Extractable work and coherence

In the previous discussions, we have shown that the work should be supplied to maintain the NESS of the system in such a way that the greater the deviations of the system from equilibrium the more the required work. Here, we in turn consider how much work can be extracted from the NESS in a thermally isolated process and reveal the role of coherence in the extraction of work. The maximum amount of work that can be extracted from a quantum system via cyclic and unitary operations is quantified by the so-called ergotropy [74]. To be specific, we consider a quantum system governed by Hamiltonian $\hat{H} = \sum_k \varepsilon_k |\varepsilon_k\rangle\langle\varepsilon_k|$, and prepared in a state $\rho = \sum_j r_j |r_j\rangle\langle r_j|$, with $\varepsilon_k \leq \varepsilon_{k+1}$ and $r_j \geq r_{j+1}$. The ergotropy can be expressed as the difference between the energy of the initial state and that of the final state with the minimum average energy through all possible unitary operations being of the form

$$\mathcal{E}(\rho) = \mathrm{tr}[\hat{H}\rho] - \mathrm{tr}\big[\hat{H}\hat{U}_{\min}\rho\hat{U}_{\min}^{\dagger}\big] = \mathrm{tr}\big[\hat{H}(\rho - P_\rho)\big], \tag{26}$$

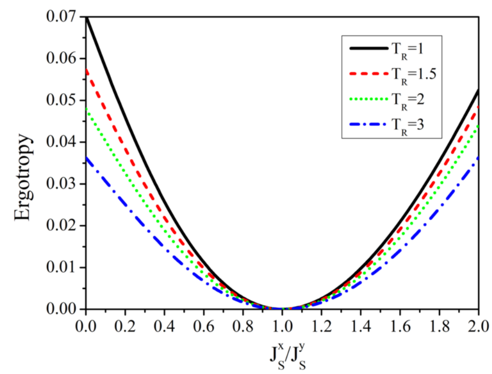

Figure 5 Ergotropy of the steady-state as a function of J_S^x/J_S^y for different temperatures of the environment. The other parameters are set as $\omega_{S_1} = \omega_{S_2} = \omega_R = 3$, $J_1 = 0.4$ and $J_2 = 1.2$

in which $P_\rho \equiv \sum_j r_j|\varepsilon_j\rangle\langle\varepsilon_j|$ represents passive state unable to generate work. By means of the explicit form of P_ρ, we obtain the well-known expression of ergotropy as

$$\mathcal{E}(\rho) = \sum_{j,k} r_j\varepsilon_k\left(\left|\langle r_j|\varepsilon_k\rangle\right|^2 - \delta_{jk}\right). \tag{27}$$

It is known that the ergotropy can only be acquired from the non-passive states. For a single-qubit system, the non-passive states could be the ones with inverted population and/or the coherence. As far as the bipartite system considered here is concerned, we are interested in the contributions of correlations induced by the common reservoir to the ergotropy. In Fig. 5, for a single common reservoir, we show ergotropy of the steady-state of the two-qubit system as a function of J_s^x/J_s^y for different T_R for the same condition as that considered in Fig. 3 (i.e., $\omega_{S_1} = \omega_{S_2} = \omega_R$, $J_1^x = J_1^y$ and $J_2^x = J_2^y$). It is obvious that the variations of ergotropy are consistent with the coherence of the system as shown in Fig. 3(a), in particular, it vanishes at the point of $J_s^x/J_s^y = 1$ at which the coherence becomes zero. Moreover, we have verified that the ergotropy of the two individual subsystems always remain zero within the whole regions of parameters so that the ergotropy can be completely attributed to the correlations of the system. In Ref. [85], Francica et al's highlighted the role of coherence in work extraction process from a quantum system by identifying a contribution to the ergotropy corresponding precisely to initial coherence of the system in the energy basis. In this sense, our results based on a particular setting of common environments are consistent to that obtained in Ref. [85]. Here, we display the synchronous variations of total ergotropy and coherence though the coherent ergotropy, i.e., part of extractable work which cannot be obtained by means of incoherent operations, is not always a coherence monotone [85].

Next, in Fig. 6(a) and (b), we further compare the ergotropy and the coherence of the system against J_1^x/J_2^x for the situation of $\omega_{S_1} = \omega_{S_2} = \omega_R$ and $J_s^x = J_s^y$ for different T_R. One can still see that the variations of ergotropy and coherence keep consistent implying the contributions of the latter to the former. In Fig. 6(a), we find the points of sudden changes of the ergotropy for $T_R = 1$ and $T_R = 1.5$, which, by the definition of ergotropy, are induced by the intersect of two eigenvalues of the system's density operator, as shown in the inset of Fig. 6(a) for the case of $T_R = 1.5$ (cf. the point labeled by the arrows).

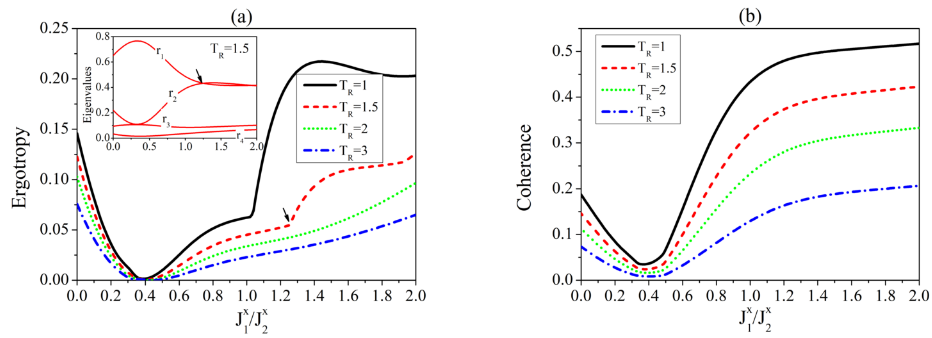

Figure 6 Ergotropy (**a**) and coherence (**b**) of the steady-state as a function of J_1^x/J_2^x for different temperatures T_R of the reservoir. The other parameters are set as $\omega_{S_1} = \omega_{S_2} = \omega_R = 3$, $J_S^x = J_S^y = 0.5$, $J_1^y = 0.4$ and $J_2^y = 1.2$

5.3 Scaling effect of multiple reservoirs

So far, the obtained results are based solely on a single common reservoir with $M = 1$. In the following, we discuss the scale effect of the reservoirs, i.e., the configuration where the subsystems S_1 and S_2 are coupled simultaneously to a cluster of M reservoirs, in the steady-state currents of work and heat, the coherence and ergotropy. To make the comparison on the same footing, we set identical frequency ω_R and temperature T_R for all the M reservoirs. By replacing $\hat{\sigma}_{R_k}^{x(y)}$ in Eq. (18) with the collective operators $\hat{S}_R^{x(y)} \equiv \sum_{k=1}^M \hat{\sigma}_{R_k}^{x(y)}$, the collective interaction between subsystem S_i with the cluster of M reservoir qubits reads

$$\hat{V}_i = J_i^x \hat{\sigma}_{S_i}^x \hat{S}_R^x + J_i^y \hat{\sigma}_{S_i}^y \hat{S}_R^y, \tag{28}$$

where we have assumed identical interaction strengths $J_i^{x(y)}$ between S_i with the M reservoir qubits. With this collective interaction, the dissipative term of the master equation and formulations of currents of heat and work remain the similar structures as that given in (20)–(24) for the interaction of S_i with a single reservoir R_k. The scale effect is embodied in the replacement of expectation value of $\hat{\sigma}_{R_k}^z$ with that of $\hat{S}_R^z$. As the state of the cluster of M reservoirs reads $\rho_R = \prod_{k=1}^M \rho_{R_k}$ with ρ_{R_k} being given in Eq. (16), the expectation value of the collective Pauli operator $\hat{S}_R^z$ can be obtained as

$$\langle \hat{S}_R^z \rangle_{\rho_R} = -M\xi_R, \tag{29}$$

where $\xi_R = \tanh(\beta_R\omega_R)$. By inserting the expression of $\langle \hat{S}_R^z \rangle_{\rho_R}$ into Eqs. (21)–(24), the local and nonlocal parts of currents of heat and work can be further derived as

$$\dot{Q}^{\text{loc}} = \sum_{i=1}^{2} \omega_R \big[-2J_i^x J_i^y \langle \hat{\sigma}_{S_i}^z \rangle_{\rho_S} - M \big((J_i^x)^2 + (J_i^y)^2 \big) \xi_R \big], \tag{30}$$

$$\dot{Q}^{\text{non-loc}} = -M\omega_R \xi_R \sum_{\substack{i,j=1 \\ (i\neq j)}}^{2} \big[J_i^x J_j^x \langle \hat{\sigma}_{S_i}^x \hat{\sigma}_{S_j}^x \rangle_{\rho_S} + J_i^y J_j^y \langle \hat{\sigma}_{S_i}^y \hat{\sigma}_{S_j}^y \rangle_{\rho_S} \big], \tag{31}$$

$$
\begin{aligned}
\dot{W}^{\mathrm{loc}} = {}& \sum_{i=1}^{2}\Big[-\big((J_i^x)^2 + (J_i^y)^2\big)\big(-M\omega_R\xi_R + \omega_{S_i}\langle\hat{\sigma}_{S_i}^z\rangle_{\rho_S}\big) \\
& + 2J_i^xJ_i^y\big(\omega_R\langle\hat{\sigma}_{S_i}^z\rangle_{\rho_S} - M\omega_{S_i}\xi_R\big)\Big] \\
& - \sum_{\substack{i,j=1\\(i\neq j)}}^{2}\Big[J_s^x\big((J_i^y)^2 + (J_j^y)^2\big)\langle\hat{\sigma}_{S_i}^x\hat{\sigma}_{S_j}^x\rangle_{\rho_S} \\
& + J_s^y\big((J_i^x)^2 + (J_j^x)^2\big)\langle\hat{\sigma}_{S_i}^y\hat{\sigma}_{S_j}^y\rangle_{\rho_S}\Big],
\end{aligned} \tag{32}
$$

and

$$
\begin{aligned}
\dot{W}^{\mathrm{non-loc}} = {}& \sum_{\substack{i,j=1\\(i\neq j)}}^{2}\Big[-M\xi_R\big\{\langle\hat{\sigma}_{S_i}^x\hat{\sigma}_{S_j}^x\rangle_{\rho_S}\big(\omega_{S_i}J_i^yJ_j^x - \omega_RJ_i^xJ_j^x\big) \\
& + \langle\hat{\sigma}_{S_i}^y\hat{\sigma}_{S_j}^y\rangle_{\rho_S}\big(\omega_{S_i}J_i^xJ_j^y - \omega_RJ_i^yJ_j^y\big) \\
& - 2\langle\hat{\sigma}_{S_i}^z\rangle_{\rho_S}\big(J_s^xJ_i^yJ_j^x + J_s^yJ_i^xJ_j^y\big)\big\} \\
& + 2\langle\hat{\sigma}_{S_i}^z\hat{\sigma}_{S_j}^z\rangle_{\rho_S}\big(J_s^xJ_i^yJ_j^y + J_s^yJ_i^xJ_j^x\big)\Big].
\end{aligned} \tag{33}
$$

The above expressions clearly show the dependence of currents of work and heat on the number M of involved reservoirs. On the one hand, the increase of M means the system collides with more reservoirs so that more work should be supplied to sustain the successive collisions. On the other hand, one might expect that the system will deviate more farther from the equilibrium in the steady-state and thus more coherence and ergotropy can be achieved. We have actually verified these results in Fig. 7(a), (b) and (c) for the currents of work and heat, the coherence and the ergotropy, respectively, for different M of the reservoirs. We observe that the increase of M leads to the rise of currents of work and heat and at the same time the growth of coherence and ergotropy. We note that in Ref. [86], it is also found that the steady-state coherence of a system (qubit) can grow with respect to the number of reservoirs coupled to it. To make the production of steady-state coherence possible, they adopted composite system-reservoirs interaction consisting of parallel and orthogonal components with respect to the Hamiltonian of the system [86]. By contrast, our model contains two qubits embedded in common reservoirs so that the coherence can also be thought of as correlations between the two qubits induced by common reservoirs. Though different in types of coherence and generating mechanisms, both models exhibit similar scaling effect of multiple reservoirs on the coherence. The underlying reason for this scaling effect is that additions of reservoirs are equivalent to the increase of coupling strengths between system and reservoirs [86–88], as can be seen from the interaction in Eq. (28).

6 Conclusion

Being distinct from independent reservoir, the common reservoir can result in some unique effects, such as dark state and steady-state coherence, which have been extensively studied in the dynamics of open quantum system and applied in some quantum technologies. In this work, we examine the quantum thermodynamics by considering a

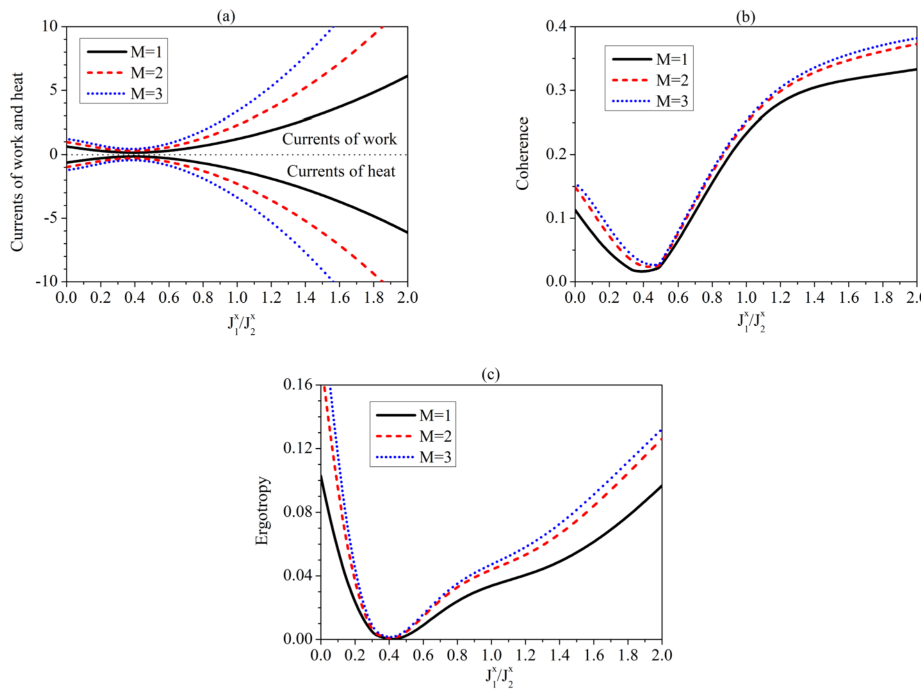

Figure 7 Currents of heat and work (**a**), coherence (**b**) and ergotropy (**c**) as a function of J_1^x/J_2^x with respect to the number M of the involved reservoirs. The other parameters are set as $T_R = 2$, $J_S^x = J_S^y = 0.5$, $\omega_{S_1} = \omega_{S_2} = \omega_R = 3$, $J_1^y = 0.4$, and $J_2^y = 1.2$

model with N coupling subsystems being surrounded by M common reservoirs. We first construct general formulations of thermodynamics quantities and particularly identify the local and nonlocal components of the total heat and work. We therefore confirm the existence of nonlocal work in the common reservoir resembling what has been found for nonlocal heat.

The steady-state behaviors of these thermodynamics quantities are demonstrated by considering two coupled qubits embedded in a common reservoir. We have shown that nonzero steady-state currents of heat and work could still emerge due to inner interactions of subsystems even when the energy conservation holds for interaction of each subsystem with the reservoir. We have exhibited the effect of dark state on system's thermodynamics and found that the more proportion of the dark state, the less the steady-state currents of heat and work.

We then explore relations between the deviation of system's steady-state from equilibrium, the work cost and the extractable work. We show that the more farther the system deviates from equilibrium (i.e., the larger the coherence and/or the temperature bias of the subsystem to the reservoir), the more work should be supplied. However, if the dark state appears, it is not right for one to simply connect the coherence of the system's steady-state to the work cost because the total coherence of the steady-state in this case is contributed by two aspects, namely, the one generated in dynamical evolution and the one contained in the initial dark state. We show that only the coherence generated in the dynamics is related to the work cost, namely, the larger the generated coherence, the more the work is required. We in turn consider the relation between the coherence of the system induced by the common reservoir with the amount of extractable work in terms of ergotropy. We show that the variations of ergotropy are consistent with the coherence implying the possible transformation of coherence into useful work. We finally examine the scale effect of the reservoirs on the currents of work and heat, the coherence and the ergotropy. It turns out that an increase of number of the reservoirs need more work to be invested but can induce more coherence and thus more ergotropy. Our results reveal several unique features of quantum thermodynamics in common reservoirs which would be useful in designing quantum thermal machines and devices.

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Abbreviations
OQS, Open Quantum System; QT, Quantum Thermodynamics; QME, Quantum Master Equation; CM, Collision Model; NESS, Nonequilibrium Steady-state.

Availability of data and materials
Not applicable.

Declarations

Competing interests
The authors declare no competing interests.

Author contributions
Z.X. and R.H. derived the theoretical results; R.H. simulated the theoretical results; Z.X. wrote the main manuscript text; Y.J. and Y.J. participated in the discussion about the results. All authors reviewed the manuscript.

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