



Enhancing quantum teleportation fidelity under decoherence via weak measurement with flips

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Abstract

Noiseless quantum channels are critical to share a pure maximally entangled state for performing an ideal teleportation protocol. However, in reality the shared entanglement severely degraded due to decoherence. In this paper, we propose a quantum teleportation channel protection scheme to enhance the teleportation fidelity in presence of decoherence. Before the entangled pair enters the decoherence channel, the weak measurement and flip operations are applied to transfer the qubit to a more robust state to the effects of the noise. After the decoherence channel the reversed flip operations and weak measurement reversal are applied to recover the initial state. We illustrate our protected teleportation scheme and compare it with a protocol based on weak measurement reversal. The numerical results show that the average teleportation fidelity of our proposed scheme can be significantly improved. Although the proposed entanglement protection scheme is probabilistic, after a successful entanglement transmission, we use the standard teleportation protocol which has probability one.

Keywords: Quantum teleportation; Entanglement protection; Weak measurement; Teleportation fidelity; Decoherence channel

1 Introduction

Quantum teleportation plays a critical role in quantum communication and quantum computation networks [1–5]. Teleportation was originally proposed by Bennett et al. [6] and has been implemented in various physical platforms [7–12]. Quantum teleportation is the process of transmitting an unknown quantum state from a sender (Alice) to a receiver (Bob) using an entangled quantum channel. The quantum channel that connects Alice and Bob is an entangled pair which is the key ingredient in quantum teleportation [13]. However, in realistic implementations, noise is affecting the entangled state during its transmission to Alice and Bob which degrades the performance of the teleportation seriously [14]. One strategy to overcome the effects of noise is by modifying the standard teleportation protocol which is changing the unitary operation applied by Bob [15–20]. Another method is called distillation which offers a probabilistic method of preparing a pair of qubits with an increased amount of entanglement by using several copies of non

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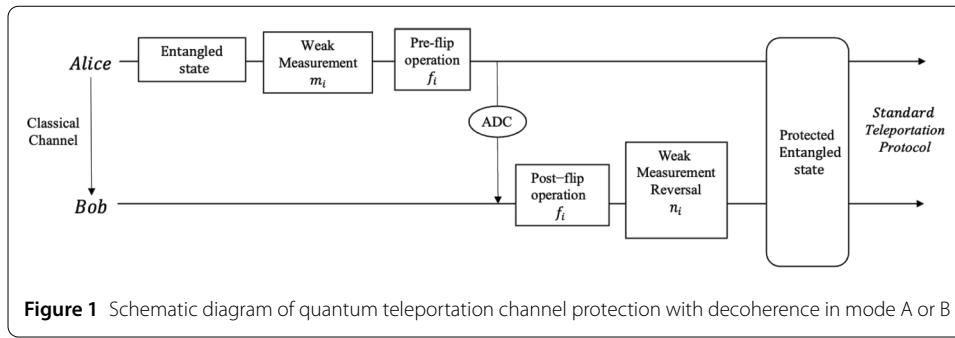
maximally entangled mixed state in conjunction with local operations and classical communication [21–23]. Applying quantum error correction also protects the entangled state through the noisy channel but requires many qubits in entanglement [17, 24–26]. Except for the previous schemes which are resource-intensive, a more recent approach to battle against decoherence is using the quantum weak measurement (WM) [27–32]. WM can decrease the disturbance of the system by weakening the interaction responsible for the measurement. There is a tradeoff between information gain and disturbance of the system in quantum WM. The weaker measurement gives less information about the system and disturb it less. Moreover, WM has been implemented in superconducting phase qubits [33, 34], as well as photonic qubits [35, 36]. In these schemes, before entering the noisy channel the state is transferred to a more robust state by a WM. After passing through the noisy channel, another WM is applied to reverse back the state to its original one.

In this paper we propose a protection scheme by WM with flips and its reversal (WMFR) to protect the entangled pair in amplitude damping channel (ADC) to enhance the teleportation fidelity. Before the qubit goes through the noisy channel, we add pre-flip operations according to the result of the WM. The aim of applying pre-flip operations is to intentionally drive the qubit close to its ground state before the ADC, to make it almost immune to the effects of the noise. After the entangled pair passes through the ADC, the post-flip operations and weak measurement reversal (WMR) are applied to recover the state. We have considered two scenarios: I. Decoherence happens in mode A or B, when the entangled pair is prepared by Alice and one qubit of the pair is sent to Bob through the noisy channel. II. Decoherence happens in mode AB, when the entangled pair is prepared by a third party (Charlie) and then each half of the entangled pair is sent to Alice and Bob through the noisy channel. For comparison, we also study another scheme which is used WM in the last step of the teleportation protocol to overcome the effects of noise, which we call it weak measurement reversal teleportation protocol (WMRTP) hereafter [19]. In this protocol, the designed joint measurement and the corresponding single-qubit reversing operation are applied to achieve optimal quantum teleportation. By comparing our scheme with the WMRTP, we show significant improvement in the teleportation fidelity of our scheme. In addition, according to the incompleteness of the WMR, the WMRTP is probabilistic and can gain high fidelity with the price of decreasing the teleportation success probability. While, our scheme has the privileged of the unit teleportation success probability. Once the protected entangled pair is successfully transferred to Alice and Bob, teleportation can be done with unit probability and high fidelity.

2 Protected teleportation with decoherence in mode A or B

In this section we assume that the decoherence happens in mode A or B, which means the entangled qubit pair is prepared by Alice (Bob) and then one qubit of the pair is sent to Bob (Alice) through an ADC, respectively. To protect the entangled pair from noisy channel, Alice applies WM and pre-flip operations on Bob's qubit before she sends it to Bob. She sends the result of the measurement to Bob through the same classical channel that she is going to use for teleportation. Then according to the result of the measurement sent by Alice, Bob applies post-flip operations and WMR to recover his qubit of the entangled pair. The schematic diagram of quantum teleportation channel protection is shown in Fig. 1.

The details of the scheme for protecting the shared entanglement state is demonstrated as follows. To establish the quantum channel, Alice prepares the entangled qubit pair and



supposes to send one qubit of the pair to Bob through an ADC. The entangled state prepared by Alice is presented as

$$|\psi\rangle_{ab} = \cos \frac{\theta}{2} |0\rangle_a |0\rangle_b + \sin \frac{\theta}{2} |1\rangle_a |1\rangle_b, \quad (1)$$

where $0 \leq \theta \leq \frac{\pi}{2}$. When $\theta = \frac{\pi}{2}$ ($\theta = 0$), the entangled pair is maximally entangled state (product state), respectively. A useful measure of the entanglement is the concurrence of this state, which is $C = \sin \theta$.

The density matrix representation of the entangled state is

$$\rho_{ab} = \begin{pmatrix} \cos^2 \frac{\theta}{2} & 0 & 0 & \cos \frac{\theta}{2} \sin \frac{\theta}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cos \frac{\theta}{2} \sin \frac{\theta}{2} & 0 & 0 & \sin^2 \frac{\theta}{2} \end{pmatrix}. \quad (2)$$

In this paper we assume that the prepared entangle pair is a maximally entangled state ($\theta = \frac{\pi}{2}$).

To protect the entangled state, Alice applies operations on the qubit of the pair which she is supposed to send to Bob, before it enters the ADC. First of all, she measures the qubit with a family of positive operator-valued measurement (POVM) consists of two operators given by $\Pi_i = m_i^\dagger m_i$ ($i = 0, 1$). The measurement operators m_i are defined as [37, 38]

$$m_0 = \begin{bmatrix} \cos(\omega/2) & 0 \\ 0 & \sin(\omega/2) \end{bmatrix}, \quad (3)$$

$$m_1 = \begin{bmatrix} \sin(\omega/2) & 0 \\ 0 & \cos(\omega/2) \end{bmatrix},$$

where $0 \leq \omega \leq \pi/2$ is the strength of the measurement. The larger ω is, the weaker the measurement becomes. When $\omega = \pi/2$ ($\omega = 0$), we have no measurement (projective measurement) and for $0 < \omega < \pi/2$ the measurement is called weak measurement.

Therefore, the applied two-qubit measurement operators are represented as

$$M_0 = I \otimes m_0, \quad M_1 = I \otimes m_1, \quad (4)$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity operator.

According to the result of the measurement, the pre-flip operations are applied on the second qubit as

$$f_0 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad f_1 = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad (5)$$

where $f_0(f_1)$ corresponds to the measurement outcome $m_0(m_1)$, respectively.

If the result corresponds to measurement operator m_0 acquired, it means the qubit is already close to the ground state $|0\rangle$ and we do not need to change it. However, if the result corresponds to m_1 appeared, by applying flip operation f_1 the qubit will be driven to its ground state to become less vulnerable to the effects of the ADC.

The two-qubit representation of the flip operations are

$$F_0 = I \otimes f_0, \quad F_1 = I \otimes f_1. \quad (6)$$

Then Alice sends the second qubit to Bob through the ADC with Kraus operators [39]

$$e_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-r} \end{bmatrix}, \quad e_1 = \begin{bmatrix} 0 & \sqrt{r} \\ 0 & 0 \end{bmatrix}, \quad (7)$$

where $0 \leq r \leq 1$ is the decaying rate.

Since only the second half of the pair passes through the ADC, the applied Kraus operators for entangled pair is

$$E_0 = I \otimes e_0, \quad E_1 = I \otimes e_1. \quad (8)$$

Also, she sends the result of the measurement 0 or 1 to Bob through the same classical channel which she is going to use for teleportation.

After receiving the state by Bob and learning about the measurement outcome, he applies the post-flip operations according to the result of Alice's measurement. The post-flip operations are same as the ones applied by Alice in Eq. (5) and (6).

Finally, Bob must apply the WMR to recover his qubit of the pair. Hence, the WMR operator should be designed in a way that $m_i n_i$ becomes almost proportionate to I . The WMR operators n_0 and n_1 are from the complete measurement sets $\{n_0, \bar{n}_0\}$ and $\{n_1, \bar{n}_1\}$, respectively, as

$$\begin{aligned} n_0 &= \begin{bmatrix} q & 0 \\ 0 & 1 \end{bmatrix}, & \bar{n}_0 &= \begin{bmatrix} \sqrt{1-q^2} & 0 \\ 0 & 0 \end{bmatrix}, \\ n_1 &= \begin{bmatrix} 1 & 0 \\ 0 & q \end{bmatrix}, & \bar{n}_1 &= \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{1-q^2} \end{bmatrix}, \end{aligned} \quad (9)$$

where $0 \leq q \leq 1$ is the strength of the WMR and $n_0^\dagger n_0 + \bar{n}_0^\dagger \bar{n}_0 = I$ and $n_1^\dagger n_1 + \bar{n}_1^\dagger \bar{n}_1 = I$. In our protection process, we only preserve the result of n_i , discard the result of \bar{n}_i and normalize the final state at the end of protection process.

The two-qubit representation of the WMR is given as

$$N_0 = I \otimes n_0, \quad N_1 = I \otimes n_1. \quad (10)$$

After applying the WMR by Bob, the entangled state shared between Alice and Bob is ready to start the teleportation protocol.

The protected entangled pair after the whole process of protection is described as

$$\rho_{ab}^{\text{fin}} = \sum_{i=0,1} \sum_{j=0,1} N_i F_i E_j F_i M_i \rho_{ab} M_i^\dagger F_i^\dagger E_j^\dagger F_i^\dagger N_i^\dagger. \quad (11)$$

According to Eq. (2) and (11), the protected entangled state, shared between Alice and Bob, after the protection process becomes

$$\rho_{ab}^{\text{fin}} = \frac{1}{2} \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix} \quad (12)$$

with $\rho_{11} = \rho_{44} = q^2 \cos^2 \frac{\omega}{2} + (1-r) \sin^2 \frac{\omega}{2}$, $\rho_{22} = \rho_{33} = r q^2 \sin^2 \frac{\omega}{2}$ and $\rho_{14} = \rho_{41}^\dagger = \sin \omega q \sqrt{1-r}$.

Since some of the results of the WMR in Eq. (9) are discarded, the protection process becomes probabilistic with total protection success probability:

$$g_{ab}^{\text{fin}} = \text{Tr}(\rho_{ab}^{\text{fin}}) = (1-r)(1-q^2) \sin \frac{\omega}{2} + q^2. \quad (13)$$

Now the entangled pair shared between Alice and Bob is ready to start the standard teleportation process. In what follows we present the standard teleportation protocol by considering the protected entangled pair Eq. (12).

Alice has the qubit ρ_{in} which we call it input state hereafter, that she wishes to teleport to Bob. The input qubit is given by

$$\rho_{\text{in}} = |\psi_{\text{in}}\rangle \langle \psi_{\text{in}}| = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}, \quad (14)$$

where $|\alpha|^2 + |\beta|^2 = 1$ and $*$ denotes complex conjugation.

Alice interacts the qubit ρ_{in} to her half of the protected entangled pair in Eq. (12). Then she makes a projective measurement on her two qubits (the input state and her share of the protected entangle state) with measurement operators $B_i = |b_i\rangle \langle b_i|$, where B_i ($i = 1, 2, 3, 4$) are represented as

$$\begin{aligned} B_i &= |b_i\rangle \langle b_i| \quad \text{with} \\ |b_1\rangle &= \cos \frac{\varphi}{2} |00\rangle + \sin \frac{\varphi}{2} |11\rangle, \\ |b_2\rangle &= \sin \frac{\varphi}{2} |00\rangle - \cos \frac{\varphi}{2} |11\rangle, \\ |b_3\rangle &= \cos \frac{\varphi}{2} |01\rangle + \sin \frac{\varphi}{2} |10\rangle, \\ |b_4\rangle &= \sin \frac{\varphi}{2} |01\rangle - \cos \frac{\varphi}{2} |10\rangle \end{aligned} \quad (15)$$

which becomes the Bell basis when $\varphi = \frac{\pi}{2}$. In this paper we use the Bell basis measurement in the teleportation protocol.

In the next step, Alice tells Bob, through the classical channel, which $|b_i\rangle$ she measured. After receiving this information, Bob knows that his state is now described as

$$\rho_{B_i} = \frac{\text{Tr}_{12}[B_i \rho_{ab}^{\text{fin}} B_i]}{g_{B_i}}, \quad (16)$$

where B_i is defined in Eq. (15) and Tr_{12} denotes the partial trace on qubits 1 and 2 (those with Alice) and $g_{B_i} = \text{Tr}[B_i \rho_{ab}^{\text{fin}}]$ the probability of gaining the measurement outcome corresponds to the measurement operator B_i .

In the last step, Bob applies the reversing operation on his qubit to recover the teleported state. The reversing operators for each measurement outcome $i = 1, 2, 3, 4$ are given as

$$R_1 = I, \quad R_2 = \sigma_z, \quad R_3 = \sigma_x, \quad R_4 = \sigma_z \sigma_x, \quad (17)$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity operator and $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are the Pauli operators.

Therefore, the final recovered state received by Bob is

$$\rho_{R_i} = \frac{R_i \text{Tr}_{12}[B_i \rho_{ab}^{\text{fin}} B_i] R_i}{g_{B_i}}. \quad (18)$$

The teleportation fidelity for each measurement outcome is defined by

$$\text{fid}_i = \langle \psi_{\text{in}} | \rho_{R_i} | \psi_{\text{in}} \rangle, \quad (19)$$

where $\rho_{\text{in}} = |\psi_{\text{in}}\rangle \langle \psi_{\text{in}}|$ is the input state which Alice wished to send, defined in Eq. (14) and ρ_{R_i} is the received state by Bob in Eq. (19).

Since the probability of occurring each state ρ_{R_i} is different, the total teleportation fidelity is defined as

$$\text{Fid}_{\text{tot}} = \sum_i g_{B_i} \text{fid}_i, \quad (20)$$

where fid_i is the fidelity corresponds to each measurement and g_{B_i} the probability of occurrence of each measurement outcome B_i .

The fidelity depends on the input state ρ_{in} . Hence, to quantify the performance of the protocol in a way that is independent of a particular input state, we use the average teleportation fidelity over all possible initial states as

$$\begin{aligned} & \langle \text{Fid}_{\text{tot}} \rangle_{\text{WMFR}} \\ &= \int d\psi \sum_i g_{B_i} \langle \psi_a | \rho_{R_i} | \psi_a \rangle \\ &= \frac{(11q^2 + \frac{q^2(11(r+q^2-1)-4q^2r)}{(q^2-1)(r-1)} + 4q \sin(w) \sqrt{1-r})}{(15(\sin^2(\frac{w}{2})(1-q^2))(1-r) - 15q^2)} - \frac{11(r+q^2-1) - 4q^2r}{15(q^2-1)(r-1)}. \end{aligned} \quad (21)$$

Also, for comparison we include the average teleportation fidelity of WM RTP proposed in [19]:

$$\begin{aligned} \langle \text{Fid}_{\text{tot}} \rangle_{\text{WM RTP}} = & \int d\psi \frac{1 + r|\alpha|^2|\beta|^2 \tan^2(\frac{\theta}{2})}{1 + r|\beta|^2 \tan^2(\frac{\theta}{2})} \left(|\alpha|^2 \cos^2\left(\frac{\theta}{2}\right) + |\beta|^2 \sin^2\left(\frac{\theta}{2}\right) \right) \\ & + \frac{1 + r|\alpha|^2|\beta|^2 \tan^2(\frac{\theta}{2})}{1 + r|\alpha|^2 \tan^2(\frac{\theta}{2})} \left(|\alpha|^2 \sin^2\left(\frac{\theta}{2}\right) + |\beta|^2 \cos^2\left(\frac{\theta}{2}\right) \right), \end{aligned} \quad (22)$$

where α and β are the elements of the input qubit density matrix in Eq. (14), θ is the angle of shared entangled state in Eq. (1) and r is the decaying rate of the ADC in Eq. (7).

And the standard teleportation protocol without any protection with the average teleportation fidelity of:

$$\langle \text{Fid}_{\text{tot}} \rangle_{\text{stan}} = \frac{1}{15} \left(4\sin(\theta)\sqrt{1-r} - 7r\sin^2\left(\frac{\theta}{2}\right) + 11 \right), \quad (23)$$

where θ is the angle of shared entangled state in Eq. (1) and r is the decaying rate of the ADC in Eq. (7).

In the following, we study the behavior of the average teleportation fidelity and total protection success probability by numerical simulations. We assume that Alice prepares a maximally entangled state with $\theta = \frac{\pi}{2}$ and the Bell basis measurement is applied in teleportation protocol with $\varphi = \frac{\pi}{2}$ in Eq. (15).

The average teleportation fidelity by varying measurement strength $0 \leq \omega \leq \pi/2$ and WMR strength $0 \leq q \leq 1$ for decaying rate $r = 0.5$ is given in Fig. 2. The gray plane in Fig. 2(a) is the standard teleportation fidelity without any protection in Eq. (23). We only accept the measurement strengths which gains higher teleportation fidelity than no protection protocol. Therefore, in Fig. 2(b) the gray area is the protection success probability with the amounts of measurement and WMR strength (ω, q) that leads to teleportation fidelity less or equal to no protection protocol.

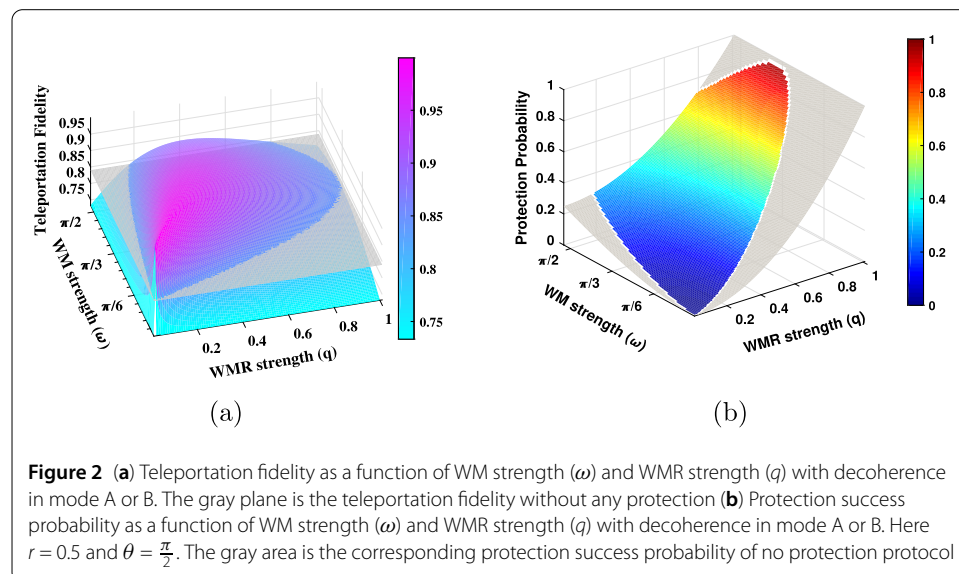
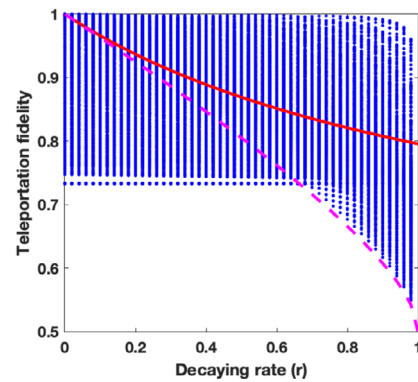


Figure 3 Teleportation fidelity by varying decaying rate $0 \leq r \leq 1$. The red solid curve represents the average teleportation fidelity of WM RTP and the dashed magenta curve corresponds to average teleportation fidelity of standard teleportation without any protection



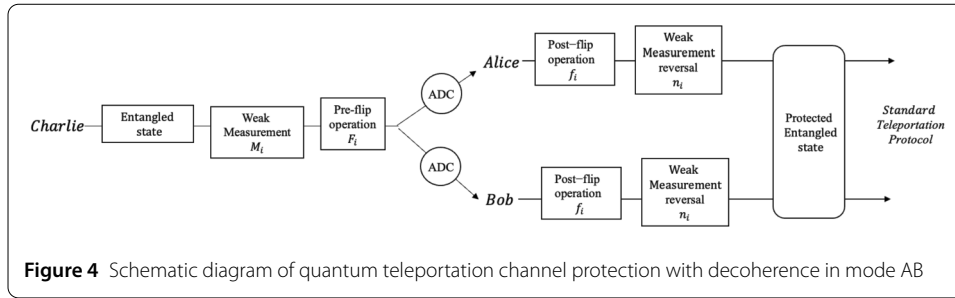
As one can see from Fig. 2 for bigger teleportation fidelities the protection success probability becomes smaller. In other words, there is a trade-off between teleportation fidelity and protection success probability. For instance, when $q = 0.53$ and $\omega = \frac{\pi}{3}$ the teleportation fidelity and protection success probability amounts are $\langle \text{Fid}_{\text{tot}} \rangle = 0.92$, $g_{\text{ab}}^{\text{fin}} = 0.34$, respectively. However, if smaller success probabilities are tolerable during protection, our scheme can gain teleportation fidelity more than 98%. Therefore, one needs to find a suitable amount for ω and q to gain acceptable teleportation fidelity and protection success probability. We note that the success probability of teleportation is always equal one, and the given probability in Fig. 2 is the success probability of the entanglement protection.

For different values of decaying rate r , each group of WM and WMR strength (ω, q) uniquely determines its corresponding teleportation fidelity. In Fig. 3 for each amount of decaying rate $0 \leq r \leq 1$, the measurement strengths (ω, q) are independently taken over all the real numbers in their range $0 \leq \omega \leq \pi/2$, $0 \leq q \leq 1$ and the corresponding teleportation fidelity is plotted.

The red solid curve represents the average teleportation fidelity of WM RTP in Eq. (22) and the dashed magenta curve corresponds to the average teleportation fidelity of standard teleportation without any protection in Eq. (23). As Fig. 3 clearly depicts, our scheme can improve the teleportation fidelity significantly compared to the standard protocol without any protection and WM RTP. Moreover, our scheme has high fidelity even under strong decoherence that would otherwise preclude any quantum operations. We note that the WM RTP is a probabilistic protocol which gains high teleportation fidelity at the price of low success probability, while in our scheme, once the entanglement protection is successful, the standard teleportation protocol with unit probability is applied.

3 Protected teleportation with decoherence in mode AB

In this section, we assume that the decoherence happens to both qubits of the entangled pair. This scenario happens when the entangled state is prepared by a third-party Charlie and then transferred to Alice and Bob via noisy channels. To protect the quantum teleportation channel, Charlie applies WM and flip operations on both qubits of the entangled pair before sending them to Alice and Bob. Also, he sends the results of the measurements. Then Alice and Bob apply the post-flip and WMR operations according to the result of the measurement of the Charlie to recover their qubit of the entangled pair. The schematic diagram of protection with decoherence in mode AB is given in Fig. 4.



In this scenario the applied two-qubit protection operators are represented as

$$X_0 = x_0 \otimes x_0, \quad X_1 = x_0 \otimes x_1, \quad X_2 = x_1 \otimes x_0, \quad X_3 = x_1 \otimes x_1, \quad (24)$$

where x_i , $i = 0, 1$ are the WM operators in Eq. (3), flip operators in Eq. (5), ADC Kraus operators in Eq. (7) and WMR operators in Eq. (10).

Therefore, the protected entangled pair after the whole process of protection is defined as

$$\rho_{ab}^{\text{fin}} = \sum_{i=0,3} \sum_{j=0,3} R_i F_i E_j F_i M_i \rho_{ab} M_i^\dagger F_i^\dagger E_j^\dagger F_i^\dagger R_i^\dagger. \quad (25)$$

With total protection success probability:

$$\begin{aligned} g_{ab}^{\text{fin}} &= \text{Tr}(\rho_{ab}^{\text{fin}}) \\ &= \sin^4\left(\frac{\omega}{2}\right)(1-r)^2 + q^4 \left[\cos^4\left(\frac{\omega}{2}\right) + r^2 \sin^4\left(\frac{\omega}{2}\right) \right] \\ &\quad + \frac{1}{2} q^2 \sin^2\left(\frac{\omega}{2}\right)(1-r) + \frac{1}{2} r q^4 \sin^2(\omega) + 2r(1-r) q^2 \sin^4\left(\frac{\omega}{2}\right). \end{aligned} \quad (26)$$

In this scenario the explicit formula of average teleportation fidelity is derived as

$$\langle \text{Fid}_{\text{tot}} \rangle_{\text{WMFR}} = \frac{\text{Top}}{\text{Down}}, \quad (27)$$

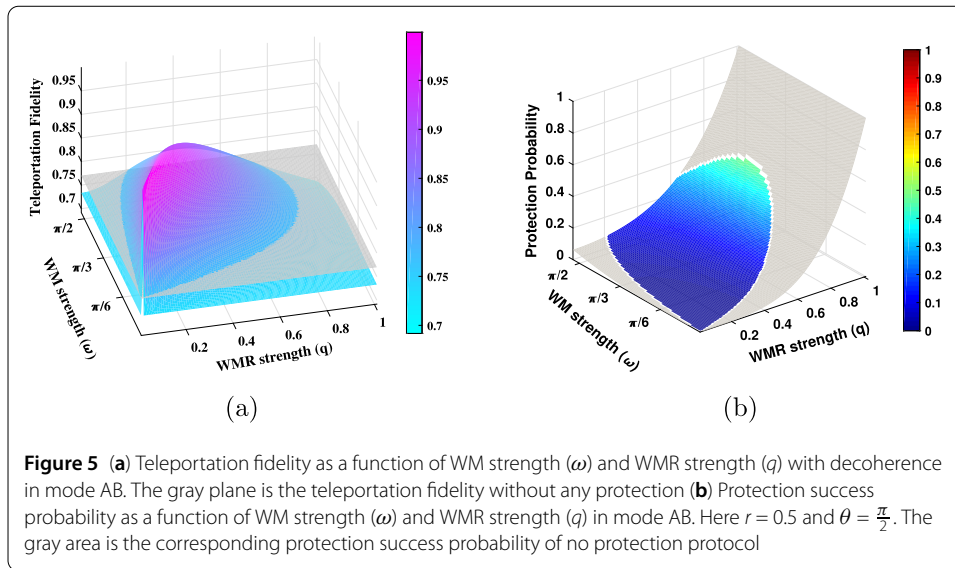
where $\text{Top} = \sin^4(\frac{\omega}{2})[22r^2q^4 - 16r^2q^2 + 22r^2 + 16rq^2 - 44r + 22q^4 + 22] + 4rq^4\sin^2(\omega) - 19rq^2\sin^2(\omega) - 44q^2\sin^2(\frac{\omega}{2}) + 22q^4 + 19q^2\sin^2(\omega)$ and $\text{Down} = 15[\sin^4(\frac{\omega}{2})(2r^2q^4 - 2r^2q^2 + 2r^2 + 4rq^2 - 4r + 2q^4 + 2)] + rq^4\sin^2(\omega) - rq^2\sin^2(\omega) - 4q^4\sin^2(\frac{\omega}{2}) + 2q^4 + q^2\sin^2(\omega)$.

For comparison we study the standard teleportation with no protection in presence of decoherence in mode AB with average teleportation fidelity of

$$\langle \text{Fid}_{\text{tot}} \rangle_{\text{stan}} = \frac{1}{15} \left(14r\sin^2\left(\frac{\theta}{2}\right)(r-1) + 4\sin(\theta)(1-r) + 11 \right). \quad (28)$$

To investigate the behavior of the average teleportation fidelity and protection success probability, we plot them as a function of WM and WMR strengths (ω, q) in Fig. 5.

In Fig. 5(a) the gray plane indicates the average teleportation fidelity with no protection and in Fig. 5(b) represents the corresponding protection success probability of no protection protocol. From Fig. 5(a), it is noted that the average teleportation fidelity is indeed



improved compared to no protection teleportation. To achieve an acceptable amount of average teleportation fidelity, one needs to tolerate low protection success probability. For instance, when $q = 0.81$ and $\omega = 0.52$ the average teleportation fidelity is 0.92 with corresponding protection success probability 0.1. Here again we notice that after a successful transmission of protected entangled pair, the teleportation will be done with unit probability.

4 Conclusion

We have proposed a scheme based on WM with flip operations to enhance the teleportation fidelity in presence of decoherence. Before the entangled pair enters the decoherence channel, we apply WM and flip operations to make it almost immune to the effects of the noise. After receiving each half of the pair by Alice or Bob, they will apply post flip operations and WMR to restore their qubit. We have demonstrated that by using our scheme an unknown quantum state can be teleported with high fidelity when the decoherence happens in mode A or B and mode AB. Moreover, the proposed WMFR has better performance in dealing with high damping probabilities that would otherwise preclude any quantum operation. Also, our scheme can highly improve the teleportation fidelity compared with the teleportation protocol based on WMR proposed in [19] and the teleportation without any protection. Our scheme has applications for battling the decoherence in other quantum communication tasks, such as quantum key distribution (QKD) [40, 41], multipartite teleportation, entanglement transmission and one-way quantum repeater.

Funding

The research of JJ Nieto has been partially supported by the Agencia Estatal de Investigación (AEI) of Spain, project PID2020-113275GB-I00, co-financed by the European Fund for Regional Development (FEDER); and by Xunta de Galicia under grant ED431C 2019/02. National Natural Science Foundation of China under Grant No. 61973290.

Abbreviations

WM, Weak Measurement; WMFR, Weak Measurement with Flips and its Reversal; ADC, Amplitude Damping Channel; WMR, Weak Measurement Reversal; WM RTP, Weak measurement Reversal Teleportation Protocol.

Availability of data and materials

Not applicable. For all requests relating to the paper, please contact the first author.

Declarations

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

SH conceived and developed the idea of WMFR, performed the experiments and analyzed the results. SC conceived and supervised the project. JJN discussed the results, and commented on the manuscript. All authors read and approved the final manuscript.

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Received: 8 September 2021 Accepted: 19 May 2022 Published online: 31 May 2022

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